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# Scalarising of optimisation criteria proposal for multi-objective optimisation of ship hull structure by evolutionary algorithm

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ABSTRACT: Assuming that the multi-objective optimization methods can be primarily classified with respect to the sources of inspiration of the methods, they can be divided into two following categories: (1) classical methods, and (2) methods inspired by natural systems, evolutionary methods in the peculiarity. Evolutionary methods can be divided, considering the way the optimization criteria are accounted for, into two groups: (2.1) scalarising methods employing a substitute scalar objective function, and (2.2) methods employing the Pareto domination relation. In the case of the scalarising methods the formulation of the objective function is a key factor for efficiency of the computational algorithm. This paper presents a scalarisation technique reformulate the original multi-objective problem of ship structural optimisation into a parametric single-objective optimisation problem applicable to application in evolutionary optimisation algorithms.

## 1 INTRODUCTION

Generally speaking, ship structural design consists in selection and spatial arrangement of material in the form of structural components (decks, bulkheads, hull sections) composed of secondary structural elements (stiffeners, plating etc.). The ship hull should provide safe ship operation at the least costs.

In the previous papers Sekulski (2011a, 2011b, 2011c, 2013) the evolutionary algorithm based on the genetic algorithm procedures was proposed to the optimization of the seagoing ship structure using in the process of the selection the combined fitness function including in one mathematical expression: (1) optimization objectives, (2) penalty function for constraints violation, and (3) domination attributes (dominance rank as well as dominance count). A practical example of the application of the developed algorithm has been presented, featuring the multi-objective optimization of the structure of fast passenger-vehicle ferry concept named Auto Express 82 m.

In the cited papers the problem of formulation of the substitute scalar optimization criterion which, when fulfilling the appropriate mathematical conditions, can be taken directly as fitness function $\mathcal{F}(\mathbf{x})$ controlling the simulated evolution of the trial solutions was not addressed. This problem is therefore a motivation for the present paper.

## 2 SCALARISATION PROCEDURE FOR EVOLUTIONARY MULTI-OBJECTIVE OPTIMISATION OF SHIP STRUCTURE

### 2.1 *General*

Assuming that the multi-objective optimization methods can be primarily classified with respect to the sources of inspiration of the methods, they can be divided into two following categories: (1) classical methods, and (2) methods inspired by natural systems, evolutionary methods in the peculiarity. Evolutionary methods can be divided, considering the way the optimization criteria are accounted for, into two groups: (2.1) scalarising methods employing a substitute scalar objective function as a single-value absolute measure of quality of the generated variants of the trial solutions, and (2.2) methods employing the Pareto domination relation to classify the variants with respect to their quality. In the case of the scalarizing methods the most important point of calculation tool for multi-objective optimization of ship structure is appropriate formulation of parameterized objective function consisting in summing the optimization objectives with proper weight coefficients. The weight coefficients values of this substitute objective function are proposed by the expert corresponding to the multi-objective optimization strategy. The simplest concept is the introduction of objective function $F(\mathbf{x})$ as a linear combination of $S$ partial optimization objectives $f_s(\mathbf{x})$:

$$F(\mathbf{x}) = \sum_{s=1}^{S} w_s f_s(\mathbf{x})$$

substitute objective = optimization objectives (1)

where $w_s$ are coefficients determining the weights assigned to particular optimization objectives, $S$ is the number of objectives. The multi-objective optimization problem is in this way reduced to the substitute single-objective problem.

Formulation of the objective function in the form of substitute scalar optimization objective in form of Equation 1 is a commonly accepted practice. Employing his method for genetic algorithms, three cases should be considered dependant on the types of partial optimization criteria $f_s(\mathbf{x})$.

i. In the case of a maximization problem all partial optimization objectives, $f_s(\mathbf{x}) \rightarrow$ max!, substitute (aggregated) objective function is also maximized, $F(\mathbf{x}) \rightarrow$ max! as in the case of the genetic algorithms is consistent with solve the problem of maximization of fitness function, $\mathcal{F}(\mathbf{x}) \rightarrow$ max!, which is a measure of quality of generated solutions and directly influences probability of selection of generated individuals (variants) Back (1996), Coley (1999), Davis (1991), Goldberg (1989).
ii. In the case of a minimization problem all partial optimization objectives, $f_s(\mathbf{x}) \rightarrow$ min!, substitute (aggregated) objective function is also minimized, $F(\mathbf{x}) \rightarrow$ min!. In this case, substitute objective function $F(\mathbf{x})$ (Eq. 1) can not be directly adopted as the fitness function $\mathcal{F}(\mathbf{x})$.
iii. In practice we have to deal with mixed problems where certain criteria are maximized (max!) and others minimized (min!). We note, however, that the genetic algorithms solve the problem of maximization of fitness function, $\mathcal{F}(\mathbf{x}) \rightarrow$ max!, therefore, in the case of (ii) and (iii) due to this fact, to employ the genetic algorithms the minimization or mixed multi-objective optimization problem (1) must be transformed to the corresponding substitute single-objective maximization problem.

In the next part of the Section a proposition will be presented and discussed of transformation of the optimization problem (1). It is consisted of the following stages: formulating utility functions of optimization objectives (Subsection 2.2), formulating a penalty function (Subsection 2.3), formulating a dominance rank (Subsection 2.4), formulating a dominance count (Subsection 2.5), and formulating a combined fitness function (Subsection 2.6).

### 2.2 *Utility functions*

Partial optimization objectives appearing in Equation 1 were replaced by properly formulated utility functions of these objectives: $f_s(\mathbf{x}) \rightarrow u_s(f_s(\mathbf{x}))$:

$$u_s(\mathbf{x}) = \left(\frac{f_s(\mathbf{x})}{f_{s,\max}}\right)^{r_s} \rightarrow \text{max!} \Leftrightarrow f_s(\mathbf{x}) \rightarrow \text{max!} \quad (2a)$$

$$u_s(\mathbf{x}) = \left(\frac{f_{s,max} - f_s(\mathbf{x})}{f_{s,max}}\right)^{r_s} \rightarrow \text{max!} \Leftrightarrow f_s(\mathbf{x}) \rightarrow \text{min!} \quad (2b)$$

where $f_{s,max}$ are the greatest values of respective optimization objective anticipated in computations, $r_s$ are positive exponents of respective utility functions. The values of the utility function are dimensionless and normalized to unity, that means $u_s(f_s(\mathbf{x})) \rightarrow [0, 1]$. Mathematical form of utility functions also assures, that substitute scalar objective function is maximized for any types of optimization objectives, that means:

$$F'(\mathbf{x}) = \sum_{s=1}^{S} w_s u_s(\mathbf{x}) \rightarrow \text{max!} \quad (3)$$

The appropriate values of $f_{s,max}$ and $r_s$ are selected by the user on the basis of the test calculations to achieve the required convergence of the algorithm. The distribution of the utility function for exemplary values of parameters $f_{s,max}$ and $r_s$ is presented in Figure 1.

### 2.3 *Penalty functions*

A computer code for optimization of ship structures should allow for accounting for a series of design constraints, such as the local and overall strength. On the other hand, implementation of the genetic algorithms requires that the equivalent problem is formulated without any constraints. Observing that the genetic algorithms do not require continuity nor the existence of derivative functions, a concept of the penalty function has been employed by many researchers (Fox 1971, Ryan 1974, Reklaitis et al. 1983, Vanderplaats 1984). The augmented objective function of the unconstrained maximization problem $f(\mathbf{x})$ has been formulated as a penalty function:

$$f(\mathbf{x}) = \sum_{s=1}^{S} w_s u_s(\mathbf{x}) + \sum_{p=1}^{P} w_p P(\mathbf{x})_p^{r_p} \rightarrow \text{max!}$$

augmented objective = optimization objectives + constraints (4)

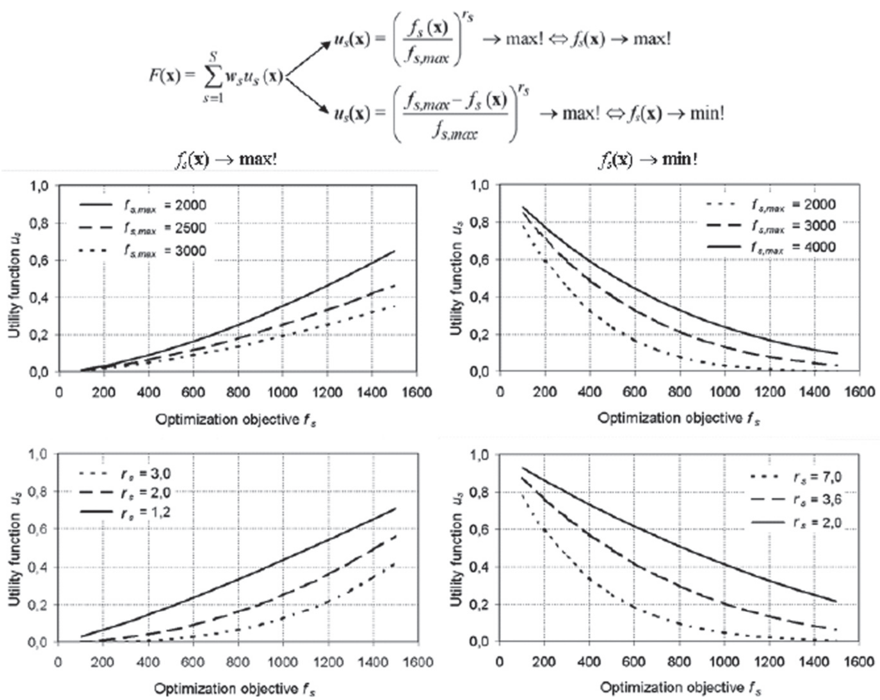


Figure 1. Graphical illustration of the concept of utility function $u_s(\mathbf{x})$ of optimization objectives $f_s(\mathbf{x})$.

where: $u_s(\mathbf{x})$—utility function of objective function $f_s(\mathbf{x})$, $S$—number of optimization objectives, $P_p(\mathbf{x})$—component of the penalty function for the violation of $p$-th constraint, $w_p$—penalty coefficient for the violation of $p$-th constraint, $r_p$—exponent of $p$-th component, $P$—number of constraints.

Mathematical form of the exponential form of the penalty functions, the same for all the constraints, was formulated so that the penalty function values $P_p(\mathbf{x})$ are dimensionless and normalized to unity. For example, in the case of rule requirement regarding main deck plate thickness the penalty function takes the form:

$$P_{13}^{r_{13}} = \begin{cases} \left(e^{\left|t - t_{rule}\right|}\right)^{r_{13,\mathrm{I}}} & \text{for } t < t_{rule} \\ \left(e^{\left(t_{rule} - t\right)^2}\right)^{r_{13,\mathrm{II}}} & \text{for } t \geq t_{rule} \end{cases} \tag{5}$$

where: $t$—actual (generated by the algorithm) plate thickness, $t_{rule}$—rule thickness. The appropriate values of $w_p$ as well as $r_p$ are selected by the user on the basis of test calculations to achieve to desired convergence of the algorithm. Distribution of the penalty function for exemplary values of parameter $r_p$ is presented in Figure 2.

The form of the components of the penalty function taken for the analysis has a very interesting interpretation with respect to scantlings of the structural elements of a ship hull. It is shown in Figure 2, for example, that values of the plate thickness $t$, less than the values required by the rules $t_{rule}$, $t - t_{rule} < 0$, are forbidden, as they do not meet the constraint. It follows that these values are not preferred (promoted, awarded) in the selection process. Individuals with the corresponding genes will be penalized with small values of the penalty function.[a] From the distribution of the penalty function it follows that possibility of survival of solutions violating the strictly formulated constraints is admissible, even though with a tiny probability. Due to this fact there is a possibility

[a]The proposed realisation of selection causes that some individuals have a chance to participate in the reproduction which slightly violate the imposed constraints but have other advantageous features. In the next generations disadvantageous features can be removed or interchanged in the operations of mutation and crossing, and the changed variant can turn to be very advantageous solution. Such realisation of selection increases the capability of searching the solution space and makes it possible to overcome "barriers" and "gulleys" in the multimodal solution space.

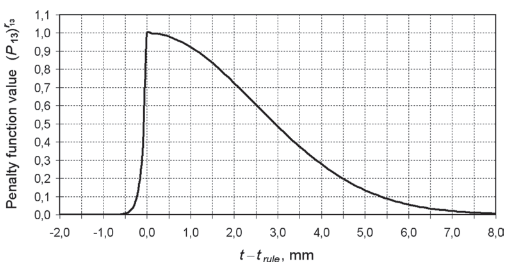


Figure 2. Graphical illustration of concept of penalty function; $r_{13} = r_{13,\mathrm{I}} = -10.0$ for $t - t_{rule} < 0$, $r_{13} = r_{13,\mathrm{II}} = -0.08$ for $t - t_{rule} \geq 0$.

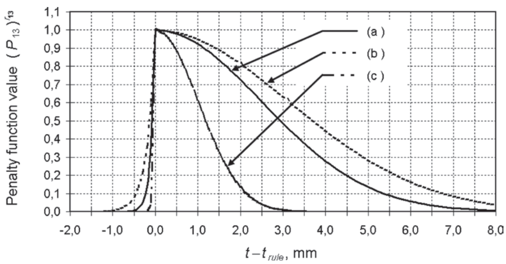


Figure 3. Illustration of influence of parameter $r_p$ of penalty function on course of function (Eq. 5): (a) $r_{13} = r_{13,\mathrm{I}} = -10.0$ for $t - t_{rule} < 0$, $r_{13} = r_{13,\mathrm{II}} = -0.08$ for $t - t_{rule} \geq 0$ (soft selection); (b) $r_{13} = r_{13,\mathrm{I}} = -5.0$ for $t - t_{rule} < 0$, $r_{13} = r_{13,\mathrm{II}} = -0.05$ for $t - t_{rule} \geq 0$ (dilution of soft selection); (c) $r_{13} = r_{13,\mathrm{I}} = -30.0$ for $t - t_{rule} < 0$, $r_{13} = r_{13,\mathrm{II}} = -0.0$ for $t - t_{rule} \geq 0$ (concentrate of soft selection).

that the small number of solutions violating the constraints will be corrected in the next generations and will transmit other advantageous features to the descendants improving the algorithm convergence. In the process of selection variants will be promoted of thickness slightly greater than required by the rules $t_{rule}$, $t - t_{rule} > 0$. The greater values of the plating thickness $t$ will be less preferred as it is not necessary to increase the plate thickness excessively considering this constraint. Such individuals will be penalized for too large, despite their admissibility, values of the plate thickness $t$; it is a small value of the corresponding component of the penalty function which is the penalty in this case.[b)]

The adopted form of the penalty function can be interpreted similarly with respect to other parameters defining dimensions of the structural elements, e.g. section modulus, moment of inertia, cross-sectional area. Possibility to change the values of exponents $r_p$ in the areas, that is $r_{p,\mathrm{I}}$ and $r_{p,\mathrm{II}}$, as well as fine fitting the shape of the penalty function (see Fig. 3) causes it to be very subtle and useful tool in controlling evolutionary optimization of structures.

As the augmented objective function $f(\mathbf{x})$ expressed by the relation Equation 4, with utility functions as well as penalty components dimensionless and normalized to unity, is: (1) defined, (2) single-valued, (3) ascending, having real values and positive in the search space, it can be adopted directly as the fitness function.

### 2.4 *Dominance rank*

The scheme of multi-objective optimization proposed in Equation 4 allows only for rough differentiation of feasible solutions with regard to domination relation in Pareto sense and does not account for information about how many solutions are dominated by a given solution.

For the solving of the mentioned problem the author proposed a scheme in which the feasible solutions are ranked by the number of other solutions dominated by them, relative to the number of feasible solutions in the current population. Therefore, dominance rank $R_{fi}$ of $i$-th feasible solution is specified by an equation:

$$R_{f\,i}(i) = \frac{\sum_{j=1,\,j\neq i}^{N_{fi}} dm(i,j)}{N_{fi}} \rightarrow \max! \qquad (6)$$

where $dm(i, j) = 1$ when $i$ dominates $j$, and $dm(i, j) = 0$ in other cases, $i, j$—indices of verified feasible solutions, $N_{fi}$ – number of feasible solutions in the current population. Advantages of the proposed strategy are: (1) ease of calculations, (2) standardization of dominance rank values in [0, 1] range, and (3) ascending values of dominance rank for solutions approaching the Pareto front (lying at the edge of feasible set). Thanks to properties (2) and (3) the value of dominance rank calculated in the proposed way may be directly included in the fitness function. In such a case selection is going to promote feasible solutions located close to the Pareto front, while the solutions lying gradually further and further from the Pareto front are going to be promoted weaker and weaker, which is a numerical realization of selection pressure exerted on the solutions located close to the Pareto front and which thus enhances the exploitation performance of the

[b)]Since so defined penalty function rewards good variants, it is natural to refer to it as a reward or preference function, rewarding or preferring good variants. Yet the common name: penalty function is used as it is done in the references.

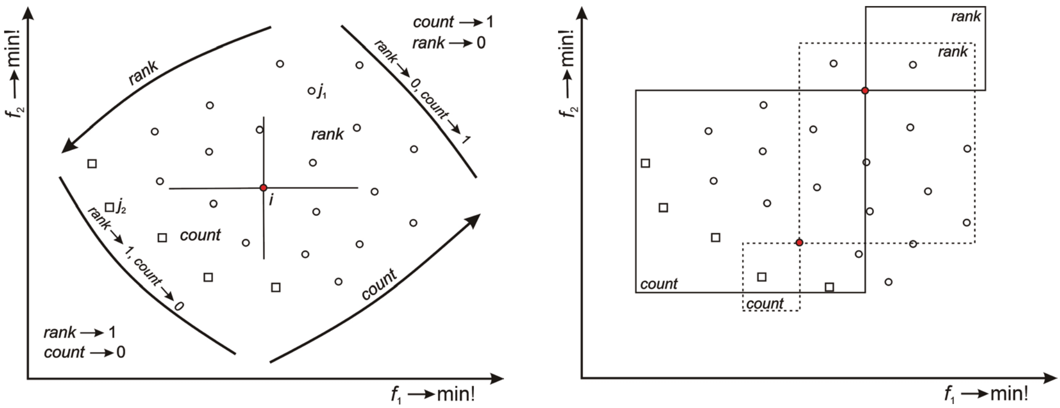


Figure 4. Graphical illustration of the dominance rank and dominance count concepts; $f_1 \rightarrow$ min!, $f_2 \rightarrow$ min!.

algorithm[c)], Figure 4. disadvantage of the proposed strategy is computational complexity $N^2$.

## 2.5 *Dominance count*

Similarly, feasible solutions can be classified by the number of solutions dominating them, relative to the number of feasible solutions. Thus, evaluation dominance count $C_{fi}$ of $i$-th feasible solution is expressed by the formula:

$$C_{fi}(i) = \frac{\sum_{j=1, j\neq i}^{N_{fi}} dm(j,i)}{N_{fi}} \rightarrow \text{max!} \tag{7}$$

where $dm(j, i) = 1$ when $j$ dominates $i$, and $dm(j, i) = 0$ otherwise, $i$, $j$—indices of verified feasible solutions, $N_{fi}$—number of feasible solutions in the current population. The dominance count defined in this way has the above mentioned properties (1) and (2), and property (3) ascending values of dominance count for the variants situated further and further from the Pareto front (located deep inside the feasible set). Thanks to properties (2) and (3) the value of dominance count calculated in the proposed way may also be directly included in the fitness function—in such a case selection is going to promote feasible solutions located far from the Pareto front, while the solutions approaching the Pareto front are going to be promoted weaker and weaker, which is a numerical realization of selection pressure exerted on solutions located far from the Pareto front and which thus enhances the exploratory properties of the algorithm, see Figure 4. Similarly as in the previous case, the disadvantage of the proposed strategy is computational complexity $N^2$.

As it has already been mentioned, the strategies for dominance ranking and the dominance count of the feasible variants proposed by the author allows for their inclusion directly in the earlier formulated (Eq. 4) extended objective function of a unconstrained maximization problem $f(\mathbf{x})$:

$$f(\mathbf{x}) = \sum_{s=1}^{S} w_s u_s(\mathbf{x}) + w_{rank} R_{fi}(\mathbf{x}) + w_{count} C_{fi}(\mathbf{x}) + \sum_{p=1}^{P} w_p P(\mathbf{x})_p^{r_p} \rightarrow \text{max!}$$

combined objective
= optimization objectives
+ dominance rank + dominance count (8)
+ constraints

where: $R_{fi}(\mathbf{x})$—dominance rank of feasible variant, $w_{rank}$—dominance rank weight coefficient, $C_{fi}(\mathbf{x})$—dominance count of feasible variant, $w_{count}$—dominance count weight coefficient. Assuming zero values of the weight coefficients, $w_{rank}$ and $w_{count}$, the user can decide whether the corresponding domination attributes are on or off.

## 2.6 *Conclusions*

As combined objective function $f(\mathbf{x})$ expressed by Eq. 8 is: (1) well defined, (2) single-valued, (3)

[c)]Two strategies of gaining and using knowledge on the solution space are used in the evolutionary algorithms: exploitation—allowing for finding many advantageous solutions located in the vicinity of local optima, thus representing local efficiency of algorithm; and exploration—allowing for investigating a large part of the estimation space to localize/identify areas which can potentially contains advantageous solutions; in these regions the local optimization algorithms can be then applied or increase exploitation properties of the algorithm.

ascending, having real values and positive in the search space, it has been adopted directly as the combined fitness function:

$$\mathcal{F}(\mathbf{x}) = \sum_{s=1}^{S} w_s u_s(\mathbf{x}) + w_{rank} R_{fi}(\mathbf{x}) + w_{count} C_{fi}(\mathbf{x}) + \sum_{p=1}^{P} w_p P(\mathbf{x})_p^{r_p} \rightarrow \max!$$

combined fitness
= optimization objectives + dominance rank
+ dominance count + constraints

(9)

Eq. 9 makes it possible to include the domination attributes to the process of selection of trial solutions. This concept is the key point of the proposed Combined Fitness Multi-Objective Genetic Algorithm (CFMOGA) applied to the multi-objective optimization of the ship hull structure Sekulski (2014).

## 3 SUMMARY

The method for formulation of the substitute optimization objective described in the present paper has been used for optimizing the ship structural design, Sekulski (2011a, 2011b, 2011c, 2013, 2014). Obtained results allow to formulate conclusions that the method is efficient and can be recommended for application in other multi-objective optimization algorithms. In each case it is necessary to perform test computations and investigate the efficiency of implemented computational procedures.

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