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Multi-objective topology and size optimization of high-speed vehicle-passenger catamaran structure by genetic algorithm

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ABSTRACT

Ship structural design has become recently an ever more important and difficult task, because it should always take into account several estimation criteria which are a crucial element of shipyard management, as the hull structural strength is one of the most important factors of overall ship safety, and the total cost of structural materials used for the construction of a ship is a significant part of her total construction cost. Simultaneously, a complete definition of the optimal structural design requires a formulation of size-topology-shape-material optimization task unifying the optimization problems from these four areas and giving an effective solution of this problem. So far, a significant progress towards a solution of this problem has not been achieved. An objective of the underlying paper was to develop an evolutionary algorithm for multi-objective optimization of both topology and scantlings of structural elements of large spatial sections of ships. In the paper an evolutionary algorithm where selection takes place based on the scalar objective function is proposed and applied to solve the problem of structural elements weight and cleaned and painted surface area on a high-speed vehicle-passenger catamaran structure with several design variables, such as plate thickness, scantlings of longitudinal stiffeners and transverse frames, and spacing between longitudinal and transversal members. The results of numerical experiments with the use of the developed algorithm are presented. They show that the proposed genetic algorithm can be an efficient multi-objective

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optimization tool for simultaneous design of the topology and sizing of ship structures.

1. Introduction

In the first, already published part of a paper Sekulski [37] a unified optimization task has been formulated for shape-material-topology-size optimization of a seagoing ship, and also a justification has been given for a formulation of such a task and solving in ship design practice. Moreover, significant and as yet unsolved difficulties have been identified in the process of formulation and solving of similar practical problems. As an illustration, author has formulated a partially unified topology-size optimization problem for an exemplary case of a structure of a fast passenger-vehicle ferry built in a catamaran configuration. The task of single-objective optimization of ship structure minimizing its weight has been formulated. A computation application has been built for the solving of thus formulated task, being based on a specialized genetic algorithm. Appropriate models of ship structure have been built and numerical experiments carried out. The obtained results have led to a conclusion that genetic algorithms may be discussed as a method allowing the solution of unified topology-size optimization problems for ship structures in early design stage.

In practice however, the design of such a complex object as seagoing ship structure is a solution of a multi-objective optimization task including many optimization criteria often counteracting each other, e.g. small hydrodynamic resistance vs. large cargo deadweight, high structure strength and reliability vs. low structural weight. This requires a more comprehensive search of solution space, without a capability to select one solution unequivocally selected as the best one, as it is in single-objective optimization tasks. This is because multi-objective optimization does not yield an unequivocal determination of a single variant proposed for further development, but a set of compromise solutions (infinite in general), which is used as a basis of taking a final design decision consisting in a selection of a solution (or solutions) to be further developed. The decision taken on the basis of calculation results is always loaded with subjectivity of judgment and the incompleteness of knowledge underlying it. These factors cannot be fully eradicated, but may only be minimized to some extent. The task is thus to provide a tool which would allow for appropriate identification of a set of “best possible compromises” or a single “best possible solution” as a result of a multi-objective seagoing ship structure design process.

Due to its high complexity, in spite of rising research and computational resources the multi-objective optimization of seagoing ship structures is still held back by a number of obstacles hindering its application in practice, and the attempts at this problem can be judged to be marginal. As we have already stated, there is no single optimal solution in case many optimization objectives are included in the analysis, and most authors assume that the outcome of a multi-objective optimization task is a set of Pareto-optimal solutions, while it is impossible to point to the objectively best one among them. Classical multi-objective optimization algorithms allow for finding in best case a single solution in a single algorithm run, which makes them unsuitable for multi-objective optimization tasks involving the determination of Pareto-optimal solution set. At the same time, the evolution-based algorithms for example allow for the determination of this set in a single algorithm run thanks to the fact that they process not single solutions, but usually large set of potential solutions which in their consecutive steps gradually evolve to a Pareto-optimal set.

The paper presents an application of multi-objective evolutionary algorithm for ship structural multi-objective optimization which is a development of an earlier presented Sekulski [37] single-objective evolutionary algorithm. Numerous details common for both algorithms and already discussed in the previous paper are going to be omitted from discussion now. The underlying work is particularly focused on a presentation of a proposed method for the estimation of fitness function for the evolving solutions, which takes into account the following factors (1) attained objectives fulfillment values, (2) degree of constraint violation. It was assumed that the single optimization criteria are components of the substitute optimization criterion included in the fitness function. The formulated constraints account for structural strength values estimated using procedures laid down in classification rules. A computational tool implementing the proposed algorithm has been developed.

A practical example of an application of the developed computational tool is presented, featuring a multi-objective optimization of a structure of fast passenger-vehicle ferry model design named Auto Express 82m. A multi-objective unified partially topology-size optimization problem for ship hull structure has been formulated. The precision of the developed computational model has been limited to a level appropriate for the preliminary phases of design process for structures of a similar type. Number of optimization objectives has been limited to two for the sake of simplicity of graphical presentation of results and their analysis. Apart from appropriate formulation of fitness function for the multi-objective optimization problem, the remaining elements of optimization model are the same as for the single-objective task formulated and solved in the preceding paper Sekulski [37].

The obtained research results enabled a detailed effectiveness analysis of the proposed evolutionary multi-objective optimization problem, employing a scalar objective function oriented on the potential applications in the construction of seagoing ships. More general observations of the characteristics of evolutionary optimization algorithms have also been made, as applied in the design of seagoing ship structures.

2. Basic concepts and numerical (calculation) techniques of multi-objective optimization

2.1. Formulation of multi-objective optimization problem

Multi-objective design problems are those where two or more criteria included are measured in different units and there is no acceptable way to transform them to a single value. In practice the multi-objective optimization problem which is selecting a best possible solution with regard to a list of evaluation criteria (optimization criteria) arises when there are targets to attain and there are various ways to attain these targets. In the design of a seagoing ship these may be for example to achieve (1) the lowest possible structural weight, and (2) the lowest costs of its manufacturing, which is pursued by appropriate selection of construction materials, and arranging the structural members spatially–these factors are decisive for the dimensions of members.

From mathematical point of view the multi-objective optimization may be defined in the general way as a procedure consisting in selecting an element of the set on the basis of relations establishing some order in this set. In ship design the elements of this set are in general the representations of particular problem solutions, such as ship structure variants, various types of ship main propulsor control ensuring that a specified aim of control is attained (e.g. lowest ship operation cost) etc. This set, called “set of possible solutions”, is a subset of a solution space V_x. As we know, the set of such solutions is limited by the introduction of various constraints and such a constrained set is than called “a set of feasible solutions Φ”. For obvious reasons the set of feasible solutions Φ is also a subset of solution space V_x, and each element of this space is a vector of design variables $x \in V_x$, Fig. 1. Solution space V_x may be a functional space or an Euclidean space, if all its co-ordinates are numbers:

$$x = [x_1, x_2, \ldots, x_i, \ldots, x_n]^T \in V_x \tag{1}$$

The case where the solution space is a n-dimensional Euclidean space $\Re^n$ is most often met in practical applications, as the description of objects in which design variables are functions leads to a significant rise in calculation complexity of the optimization process. In the underlying work the solution space is understood to be $\Re^n$ space.

The vector of optimization criteria f(x) is an operator which transforms the solution space V_x into an objective space V_f (named also the evaluation space or the quality space), which can be noted in a following way:

$$f(x) : V_x \rightarrow V_f \tag{2}$$

When constraints are accounted for, the vector of optimization criteria f(x) is an operator which transforms the feasible set Φ into a other set Φ_f, called an attainable objective set (also attainable goal set):

$$f(x) : \phi \rightarrow \phi_f \tag{3}$$

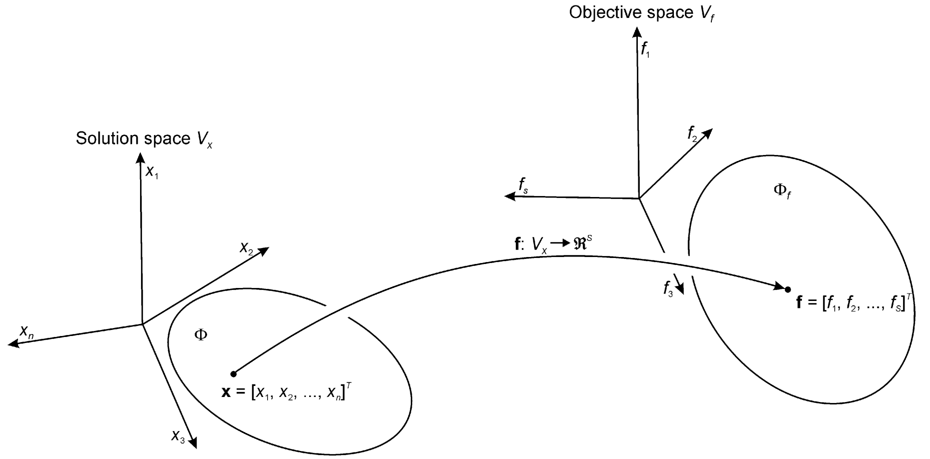

Fig. 1. Graphical illustration of multi-objective optimization task; $x \in \phi$-vector of design variables, Φ-set of feasible solutions, $f(x) \in \phi_f$-vector of optimization criteria, Φ_f-set of attainable objectives (goals, evaluations).

In case the objective space $V_f \subset \Re^s (s = 2, 3, \ldots, S)$, the operator f($x$), being a vector function consisting of S components $f = [f_1, f_2, \ldots, f_s, \ldots, f_S]^T$, assigns S numbers to each element of feasible set Φ, which taken together specify the quality of this element. We speak then of multi-objective optimization task, as the S components of vector function f(x), is S various optimization criteria. The set of attainable objectives is a subset of S-dimensional Euclidean space, and the element of this set $f(x) = [f_1(x), f_2(x), \ldots, f_s(x), \ldots, f_S(x)]^T$, called a vector of objectives (quality indices), for a fixed vector of design variables, is a set S of numbers giving the numerical values of particular quality indices. A graphical illustration of the multi-objective optimization task is presented in Fig. 1.

The goal of multi-objective optimization is highly important due to practical reasons, and at the same time not so simple as the single-objective optimization where the definition of an optimal solution is obvious. In cases where the quality of a solution is determined not by a single number but several numbers instead, the concept of multi-objective optimal solution cannot be defined using a linear order relation over the set of feasible solutions. A new concept needs to be introduced, being an analog of optimality concept for the multi-objective optimization tasks. In this regard, discussed in details in the further part of the paper, a concept of domination formulated by Pareto [29] has been widely applied.

In contrast with the way the single-objective optimization problems are solved, which consists in finding such a solution of design variable values x, that the optimization criteria $f(x)$, dependent on them reach extremum (minimum or maximum) values while staying within all the adopted constraints, the goal of a multi-objective optimization problem solving can be formulated in the following way: find a combination of design variable values x, which would optimize at the same time all components of a given objective function vector f(x). It is also possible to impose constraints on the variability ranges of design variables. It is also assumed that all the functions occurring within the problem are real ones, and the number of constraints is finite. Taking into account the demand or computational resources and their cost, another requirement may be posed, that the selection made could be implemented at the lowest possible cost. Exact definition of the meaning of word “optimize” has crucial significance in case of multi-objective optimization problem. In the further part of the paper this concept is going to be discussed in more detail. The general mathematical formulation of a multi-objective optimization problem may be as follows:

$$\textit{for design variables}\; x = [x_1, x_2, \ldots, x_i, \ldots, x_n]^T : x_{i,\min} \leq x_i \leq x_{i,\max}, \; i = 1, 2, \ldots, n, \tag{4a}$$

$$\textit{optimize}\; f(x) = [f_1(x), f_2(x), \ldots, f_s(x), \ldots, f_S(x)]^T \; s = 1, 2, \ldots, S, \tag{4b}$$

$$subject\ h_k(x) = 0\ k = 1, 2, \ldots, m_1, \tag{4c}$$

$$g_j(x) \geq 0\ j = 1, 2, m_2 \tag{4d}$$

where f(x) is a single-column objective function vector S, x is a single-column independent variable vector n (design variables), $h_k(x)$ and $g_j(x)$ are constraint functions and $x_{i,min}$ and $x_{i,max}$ – respectively and upper and lower limit of variables. The equations $h_k(x) = 0$ are called equation constraints, while the inequalities $g_j(x) \geq 0$ are called inequality constraints.

2.2. Objectives

Let's recall that objective functions representing the co-ordinates of the vector objective function $f(x) = [f_1(x), f_2(x), \ldots, f_s(x), \ldots, f_S(x)]^T \in V_f$ are functions which describe the selected characteristics of the optimized ship structure, assigning to the particular designs of this structure $x \in V_x$ real numbers[1], which constitute scales with a specific measure units, which in turn specify the quality of a given design with regard to optimization criteria $f_s(x)$. The values of objective function are a basis for the evaluation of an analyzed solution. Objective functions depend on design variables (and also optimization parameters) in an explicit or entangled way, and may be noted using mathematical expression, tables or specified using algorithms. In case of multi-objective optimization the fulfillment of optimization criteria is seeking a compromise (trade-offs) $f^c = f(x^c) = [f_1(x^c), f_2(x^c), \ldots, f_s(x^c), \ldots, f_S(x^c)]^T$, $c = 1, 2, \ldots, C$, over a set of possible solutions V_x, or over a set of feasible solutions $\Phi \subset V_x$ in case of problems including constraints, where x^c are co-ordinates of sought compromise (trade-off) solutions constituting a solution of a multi-objective optimization problem.

In the design of seagoing ship structure the optimization criteria are usually number functions $f_s(x)$ (same as for many other practical technical problems) which means that numbers are a measure of a degree to which the optimization criteria have been met, which is a quality of a solution. However, in case of a multi-objective optimization problem the formulation of a set of optimization criteria $f_s(x)$ does not allow for an unequivocal ordering of a set of possible solutions $x \in V_x$ with regard to their quality, because a single optimal solution x^* of such a problem does not exist.

Selection of optimization criteria $f_s(x)$ depend mostly on the problem solved. As, for example, the problems of economy are an integral and ever more important part of seagoing ship design process, then the most general optimization criterion which could be used for the structure of a ship would be its manufacturing cost, attainable while meeting all the technical, formal, legal and functional requirements. Attention should be paid to the fact that a general view of production cost includes the costs of design, manufacturing itself, foreseen service costs, and also the costs of decommissioning and disposing of the structure. So, the criterion formulated in this way normally appears only in the preliminary design phase but is only rarely used in further stages of detailed design work, due to a large number of design variables which would have to be accounted for in the formulation of a problem, and a difficulty encountered in the determination of relations occurring between this criterion and design variables. A series of partial structure optimization criteria is then formulated, with the following main ones among them: (1) structural weight, (2) labour demand, (3) structure manufacturing cost, (4) reliability, (5) cost of maintenance and repairs, etc. This way we approach the formulation of a multi-objective optimization problem, which is a subject of the underlying paper. It is worth noting that the value of money as a cost comparison basis is also changeable, and depends on social and economic conditions same as the mutual relations between particular components of construction/service cost. This tends to somewhat reduce the importance of cost-based criteria and leads to a situation where other criteria not expressly money-related are used more often, such as the above mentioned ones: structural weight and reliability. Quite often the volume or weight of steel materials (plates and profiles)

[1] Obviously, in practice the ship structural optimization is a discrete or, in some cases, mixed problem which can be analyzed assuming $x \in V_x$ to be integer numbers (or even natural) or a part of the set is composed of integer numbers while another part – real numbers. To keep the text within reasonable limits author consider the most general case of a continuous problem: $x \in V_x$ real numbers.

used for the fabrication of a whole structure or its part is used as a structure optimization criterion. The above criteria approximately reflect the costs of used-up material in case of structures built wholly of a single material, e.g. steel. Some other criteria can be adopted as well, reflecting the approximate labour outlay for the construction works or its maintenance works - these can be for example the volume or length of joints for a given steel structure design, or its outer surface which is subject to maintenance.

2.3. Result of multi-objective optimization: set of non-dominated (Pareto-optimal) solutions

Pareto-optimality takes its name from an Italian economist Vilfredo Pareto [29], and is a widely accepted measure of quality in the multi-objective selection problems. A particular design of a seagoing ship structure may be called Pareto-optimal under a condition that there are no exist such other variant of this structure which would be better with regard to at least one criterion while at the same time being equally good with regard to all the remaining optimization criteria. This means that a Pareto-optimal structural variant cannot be improved without simultaneous worsening of at least one criterion. Pareto-optimal designs are also referred to in literature as being non-dominated ones, trade-offs, noninferior or Pareto-efficient. The variant of a ship structure is not Pareto-optimal if there is any other variant, which improves at least one criterion while at the same time not worsening the values obtained for the remaining ones. Such variants are also called dominated ones or inferior ones.

Using the concept of domination formulated by Pareto we may state that a multi-objective optimal solution is every such a solution which has no other feasible solutions dominating it. We say that the solution x_1 dominates (is better than) the solution x_2, if:

$$f_s(x_1) \leq f_s(x_2), s = 1, 2, \ldots, S \tag{5}$$

and the inequality (5) is definite ($>$) for at least one criterion f_s. There is then no such a solution in the set of feasible ones for which the value of all criteria would be “better” than their respective values for any multi-objective optimal one. In other words, a multi-objective optimal solution is such an feasible solution, for which no better solution can be found in the set of feasible solutions. The word “better” should be understood here in the sense of Pareto domination.

Pareto-optimality concept may be presented in a form of a scatter-plot of solutions (Fig. 2) in which each objective function is represented by particular axes of coordinate system, and particular solutions are represented by graph dots. Of course, such a presentation is easy in case of problems involving two objectives, but is much more difficult of even impossible in case three or more objectives are involved. In the two-objective minimization problem Pareto-optimal solutions are those which have no solutions located below and to the left of them on the graph. Dominated solutions are those for which at least one solution lies below or to the left.

The concept of Pareto domination ($\overset{\lesssim}{\rightarrow}$ relation) allows for the introduction of a two-value measure of quality for the solutions of a multi-objective optimization problem. It allows for dividing the set of feasible solutions into two sub-sets: (1) subset of dominated solutions $\Phi_f \overset{\lesssim}{\rightarrow} \Phi_{fd}$, and (2) subset of non-dominated solutions, $\Phi_f \overset{\lesssim}{\rightarrow} \Phi_{fnd}$ which may be considered to be a solution of a multi-objective optimization problem. Two-value of this measure does not allow for a deeper evaluation of a feasible dominated solutions set[2], and in particular does not allow for a relative estimation of distances between dominated solutions and set consisting of non-dominated solutions (set of Pareto-optimal solutions) by any other feasible solution. These problems and solution suggestions will be discussed in details in the next paper prepared by the author. In spite of this, the relation of Pareto domination is the one most often used for the definition of multi-objective optimal solution. In the further part of the work, when talking about domination relation, we shall then understand it to be the relation of Pareto domination, and the earlier used phrase “optimize vector objective function f” shall be understood as a command: find the Pareto-non-dominated solutions within the feasible solutions set. Having introduced the domination relation we are then able to divide all the feasible solutions of multi-

[2] Most detailed analysis of set of feasible solutions in relation to domination. Relationship of domination does not “see” many details of the evaluation space. The only details it can “see” whether the solution is dominated or non-dominated.

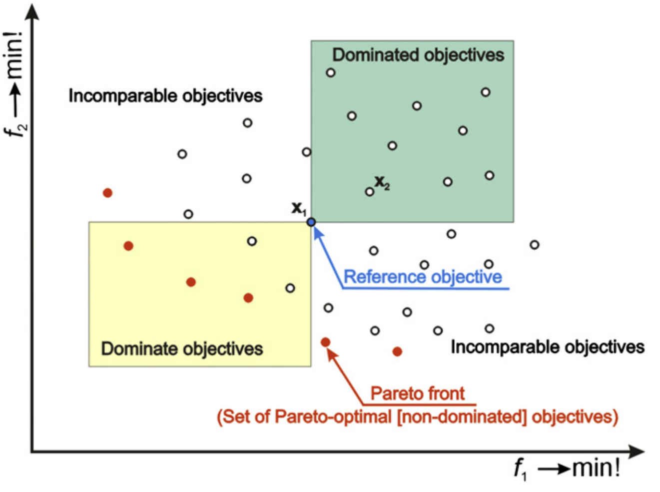

Fig. 2. Graphical illustration of possible relations between solutions in the objective space; the objectives dominated by a reference objective and the objectives dominating it are highlighted; objectives non-dominated by any other ones belonging to the set constitute a set of Pareto-optimal objectives; $f_1 \rightarrow \min!$, $f_2 \rightarrow \min!$.

objective optimization problem into two classes: (1) dominated solutions, or (2) non-dominated solutions (also called Pareto-optimal ones or compromise ones).

A basic feature of multi-objective optimal solutions is the fact that many (or even an infinite number of them) may exist for practical applications. In case an feasible set is continuous for a given problem, and is a subset of $\Re^n$, then the set of feasible objectives shall also be continuous and a subset of $\Re^2$ (as a result of two quality criteria). As a result of the analysis of feasible objectives set we may obtain not two multi-objective optimal points, but a whole curve of multi-objective optimal objectives, which is presented in Fig. 3.

The curve of non-dominated objectives in n-dimensional set of feasible objectives Φ_f corresponds to a curve of non-dominated solutions in feasible set Φ (Fig. 3). Briefly speaking, the curve of multi-objective optimal objectives is quite often called a curve of multi-objective optimal solutions or a Pareto-optimal solutions front. The nomenclature will be consequently applied in the further part of the paper.

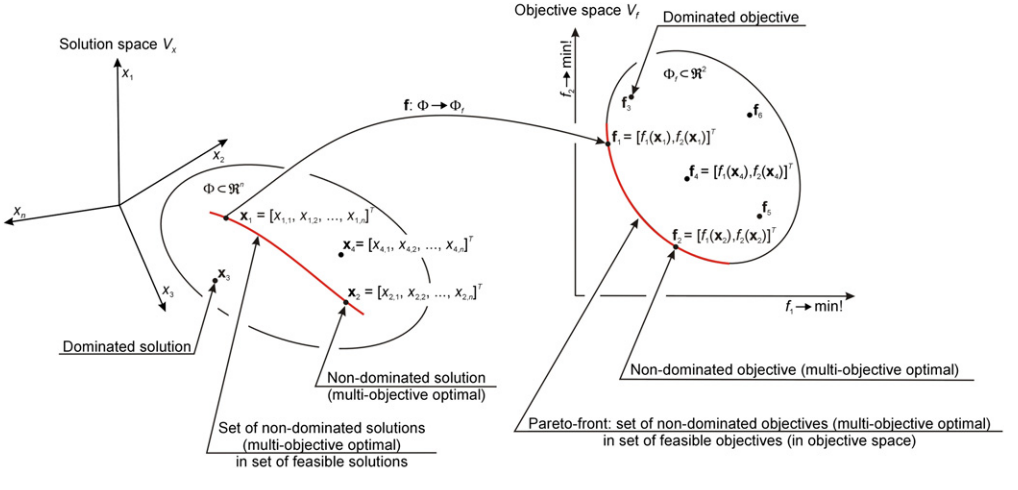

Fig. 3. Graphical illustration of line of non-dominated objectives in the objective space and set of non-dominated solutions in solution space; $f_1 \rightarrow \min!$, $f_2 \rightarrow \min!$.

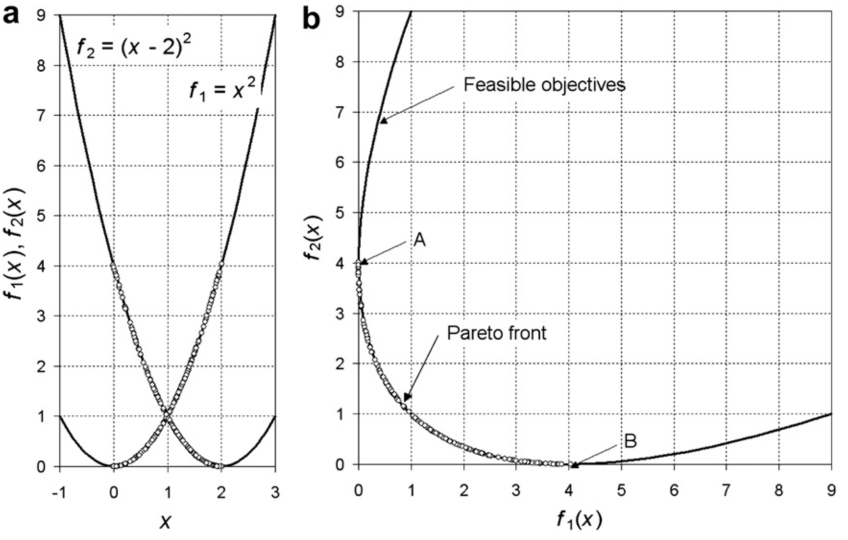

Fig. 4. Graphical illustration of evolutionary multi-objective optimization of Schaffer [36] test function: (a) set of non-dominated solutions in solution space, (b) set of non-dominated objectives in objective space; circles represent non-dominated solutions found by evolutionary algorithm.

In case the objective space $V_f = \Re^S$ (S quality criteria), then a certain S-dimensional hypersurface of multi-objective optimal solutions is obtained over S-dimensional set. Each feasible point belonging to the set of multi-objective optimal solutions is multi-objective optimal. There may be many such points, but it is impossible to establish which of them is "better".

To illustrate the concept of non-dominated solutions of multi-objective optimization problem we will present the result of the evolutionary multi-objective optimization of the well-known test function Schaffer [36]:

$$f(x) = [f_1(x), f_2(x)]^T \rightarrow \min!\left(or\, f(x) = [f_1(x) \rightarrow \min!, f_2(x) \rightarrow \min!]^T\right)$$

where:

$$f_1(x) = x^2,$$

$$f_2(x) = (x-2)^2, \ and:$$

$$x \in V_x$$

where $V_x = [-1, 3]$ is the set of feasible solutions. Courses of component functions $f_1(x)$ and $f_2(x)$ in the feasible set presented in the space of design variables as well as the set of non-dominated solutions found by the evolutionary algorithm: $x^c \in [0, 2]$ are presented in Fig. 4a. The course of the test function in the objective space as well as the found set of non-dominated solutions (Pareto front) are presented in Fig. 4b. The continuous line represents the set of all feasible evaluations for continuous problem, and the points found by the evolutionary algorithm – non-dominated evaluations being evolutionary approximation of the Pareto front. Generalizing we can say that the arc between points A and B represents the set (infinite) of the non-dominated evaluations thus being a Pareto front for the formulated optimization problem and is its solution.

2.4. Solving of multi-objective optimization problem: how find set of non-dominated solutions

Monographies and books on the general problems of multi-objective optimization are e.g.: Stadler [42], Eschenauer et al. [10], Statnikov et al. [43] and Sen & Yang [39]. Rare works focusing on multi-objective optimization of ship structure include: Shi [40], Das [5], Das et al. [6], Trincas et al. [44], Ray & Sha [32], Sen & Yang [38], Jianguo & Zuoshui [22] and Parsons & Singer [30].

The methods used for the solving of tasks of this type, outlined in monographies and currently applied in practice, my be divided into two basic groups: (1) classical methods, and (2) evolution-based methods. Cohon [4], Stadler [42], Statnikov et al. [43] discuss classical methods, which include two basic methods used for the solving of such tasks: (1) optimization problems are solved with regard to all optimization criteria taken singly one by one while the remaining criteria are included in the set of constraints, (1.2) a substitute optimization criterion is formed of the adopted ones, as a linear combination of original component criteria multiplied by appropriately selected weight coefficients, and then the optimization problem is solved with regard to such a newly formed aggregate criterion. For (1.2) case a series of calculations is usually carried out for variously adopted values of weight coefficients, and the best among the found solutions is taken as a solution of the task. The methods based on a scalarization of a vector objective function have found wide-ranging applications also in the methods of evolution-based multi-objective optimization, as they allow for use of well researched single-objective optimization algorithms. Fundamental disadvantages of methods from this group are: (a) seeking only a single point on non-dominated solutions front and a resulting necessity to make numerous calculation runs for a single optimization task, (b) sensitivity of some solutions to the shape of non-dominated solutions front, and (c) the fact that expert knowledge is required at the beginning to specify the weight coefficients used for component optimization criteria.

Classical methods used for the solving of multi-objective problems basing first of all on the scalarization of vector objective functions are easy to implement but ineffective in many cases. However, evolutionary multi-objective optimization algorithms developed in recent years have proved highly effective in this regard Deb [8], Osyczka [28], Sarker et al. [35], Abraham et al. [1], Coello et al. [3].

Not many so far but highly promising results in the field of genetic algorithms use for multi-objective optimization tasks have been obtained lately which includes also some results in the field of ship structures: Okada & Neki [27] and Hutchinson et al. [18]. Jang & Shin [20] have also applied the Evolutionary Strategy method for the multi-objective optimization of ship structures.

Special evolutionary multi-objective optimization methods may be applied, as far as genetic algorithms are concerned: VEGA – Schaffer [36], HLGA – Hajela & Lin [14], FFGA – Fonseca & Fleming [11], NPGA – Horn et al. [15], NSGA – Srinivas & Deb [41], RWGA – Murata & Ishibuchi [26], MOBES – Binh & Korn [2], SPEA – Zitzler & Thiele [48], MOMGA – Veldhiuzen [46], PAES – Knowles [23], NSGA-II – Deb et al. [9], SPEA2 – Zitzler et al. [47]. Fundamental advantages of these methods are: (1) effective search of solution space and (2) capability to illustrate the non-dominated solutions front in a single simulation run. Excellent presentation of evolutionary methods of multi-objective optimization we can find in some recently published monographies Coello et al. [3], Deb [8] and Osyczka [28].

NSGA-II Deb et al. [9] and SPEA2 Zitzler et al. [47] algorithms are commonly recognized at present and they are employed as reference algorithms by many authors for estimation of the efficiency of other formulations. The principal elements of these algorithms are: (1) selection strategies based on the Pareto-domination relation, (2) niching strategies to preserve diversity in the consecutive populations, (3) elitist strategy to ensure survival of non-dominated solutions in the time of evolution. Methods based on scalarization of the objective function are considered less effective and are a closed and no more developed stage of the history of the evolutionary multi-objective optimization. Despite it, we can, however, claim that the algorithms employing scalarization of the objective functions are efficient algorithms transient from the classic methods to the advanced algorithms employing the Pareto-domination relation for the variant selection. The researchers have reported for several years that if the number of the optimization criteria is greater than 3, the methods based on the domination relation turn to be ineffective since together with the increase of the number of optimization criteria the number of non-dominated variants decreases reducing the effectiveness of the selection operator Hughes [16], Hughes [17], Jaszkiewicz [21], Purshouse & Fleming [31]. The scalarization methods have been found promising again with the hope to: (1) developing more simple and intuitive algorithms than algorithms based on the domination relation, obtaining expertise on the multi-objective ship structural optimization, (2) developing effective algorithms for problems with a large number of the optimization criteria. Of course, it needs to be considered what number of criteria is practically justified. With regard to capability of processing data by human beings and capability to work out decisions it seems that the number of the optimization criteria in practical problems should be between 5 and 7.

Due to the following practical problems (1) lack of information about the actual localization of non-dominated solutions set, and (2) necessity to deploy significant computational resources to solve the multi-objective optimization problem, main effort in the practical evolution-based multi-objective optimization is directed at determining the acceptable approximation of Pareto set instead of accurate composition of this set. With regard to this it can be assumed that in practice the result of MOEA process is a set of non-dominated solutions called shortly the approximation of Pareto set, Fig. 5, and not the exact Pareto-optimal solutions set. Practical formulation of multi-objective optimization problem and of attained results should follow this guideline.

Finding the effective solution of a formulated problem is additionally burdent with a fact that also the determination of Pareto set approximation is a multi-objective task, because we strive for example to (1) minimize the distances between generated solutions and the Pareto set and (2) maximize the variety of solutions included in the attained approximation of a Pareto set. It must also be remembered that in practical applications the location and shape of Pareto front are unknown. It is then impossible to refer directly to this front in the evaluation of tentative solutions – it may only be expected that the sought non-dominated solutions are a good approximation of Pareto front, but this cannot be supported with any quantifiable measure. Therefore, as in any multi-objective optimization problem, also in this case it is impossible to establish exactly how good a found solution is, due to the fact that “good” is a concept of various aspects including (1) attained proximity to Pareto set (which location and shape are unknown), (2) variety of solutions included in the set approximating the Pareto front. However, this illustrates well two basic partial goals of MOEA formulation and solution finding: (1) directing the search to Pareto set, and (2) maintaining the variety among the searched non-dominated solutions, Fig. 5b.

In contrast with single-objective optimization problems, where the objective function and fitness function are often the same, the evolution-based algorithms of multi-objective optimization feature both the fitness function and the selection process taking into account a number of criteria which are included in a single fitness function. From this point of view these methods may be in general divided by the type of fitness function used for calculations into the following classes: a) selection with respect to the scalar objective function with fixed weights of optimization criteria, b) selection with respect to the scalar objective function with random weights of optimization criteria, c) division of the variant set into sub-sets and selection in each of them with respect to single criteria, Fig. 6.

First proposal of Fig. 6a, stemming from classical methods used for the determination of compromise surface, consists in summing the criteria up and formulating a single, parameterized objective function. Parameters of this substitute objective function are fixed during the optimization run, which allows for finding the one non-dominated solution. The multi-objective optimization problem is reduced to the

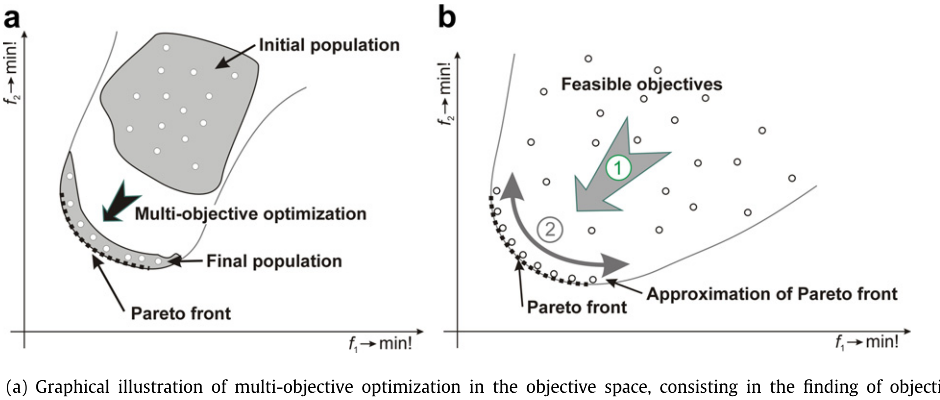

Fig. 5. (a) Graphical illustration of multi-objective optimization in the objective space, consisting in the finding of objectives (solutions) included in the Pareto-optimal solutions front, and in practice constituting a good (acceptable) approximation of Pareto-optimal solutions front (Pareto approximation); (b) graphical illustration of two basic goals of multi-objective optimization: (1) directing the search to Pareto set, and (2) maintaining the variety among the search non-dominated solutions; $f_1 \to$ min!, $f_2 \to$ min!.

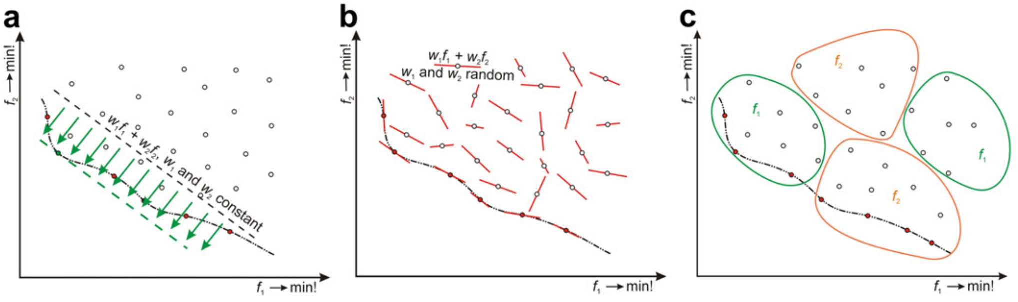

Fig. 6. Graphical illustration of selected strategies for taking into account the particular optimization criteria used in the multi-objective optimization algorithms: a) selection with respect to the scalar objective function with fixed weights of optimization criteria, b) selection with respect to the scalar objective function with random weights of optimization criteria, c) division of the variant set into sub-sets and selection in each of them with respect to single criteria; $f_1 \rightarrow \min!$, $f_2 \rightarrow \min!$.

single-objective problems, with the criteria being usually called objective functions. The simplest concept is the introduction of objective function F as a linear combination S of partial criteria f_S:

$$F = \sum_{s=1}^{S} w_S f_S, \tag{6}$$

where w_S are coefficients determining the weights given to particular criteria.

Second Fig. 6b and third Fig. 6c proposals are based on a weighted sum of optimization criteria, where weight coefficients represent the values changing in the process of evolution Hajela & Lin [14], Ishibuchi & Murata [19]. Weight coefficients of the substitute objective function change in a specific way during the optimization run, which allows for finding the non-dominated solutions set instead of a single compromise solution.

Methods based on selection with respect to the scalar objective function with random weights of optimization criteria Fig. 6b employ numerical procedures for setting random values of weight coefficients w_S. The simplest and most frequently applied implementation of the method is setting random values of uniform distribution in range 0, 1.

Methods based on selection according to single criteria as per Fig. 6c consist of mechanisms switching between the criteria during the selection phase. In each case where the algorithm commences the execution of reproduction, some criterion (potentially other) decides which member of population is going to be copied to the set of variants earmarked for crossbreeding. For example, Schaffer [36] proposed an algorithm, where the population is divided in advance to identical parts and then a different, single criterion is used on the members of each of groups one by one and Kursawe [24] proposed a different method consisting in a random selection of a single optimization criterion to be used in the next step of selection process, with probabilities to be set by the user or randomly adjusted during the evolution.

The multi-objective evolution-based optimization algorithms outlined above have been tested by other authors on simple problems of multi-objective optimization. As no systematic research into the suitability of these algorithms for the solving of optimization problems involved in the design of seagoing ship structures has been carried out so far, then the application of a particular method should be preceded by systematic research into its effectiveness in the problems involved in the design of such structures.

3. Calculation tool for evolutionary multi-objective optimization of ship structure

Genetic algorithms (GA) have already been extensively described in literature discussing their theoretical foundations, details of calculation procedures and their practical applications, so these problems are not going to be discussed here again and the reader is referred to respective literature e.g. Goldberg [13], Davis [7], Michalewicz [25].

For obvious reasons genetic algorithms are unsuitable for manual calculations, so their suitability for the solving of multi-objective optimization problems which include the optimization of topology as well as the optimization of ship structure member dimensions has been based on computer simulations. The computer software used for multi-objective optimization of ship structures has been built on the basis of a software package for single-objective optimization, presented in the preceding paper Sekulski [37], while supplementing it with a series of calculation procedures providing for multi-objective optimization. The most important of them are the procedures for: (1) analysis of feasible solutions set with regard to dominance relation, (2) control of domination, including the building of non-dominated solutions set approximating the Pareto set, and (3) calculation of fitness function values. Block-type diagram presenting the main concepts of a developed calculation software is shown in Fig. 7.

The program carries out the calculations automatically, starting from an input data set prepared by the user, which has been supplemented with the following data with regard to its single-objective

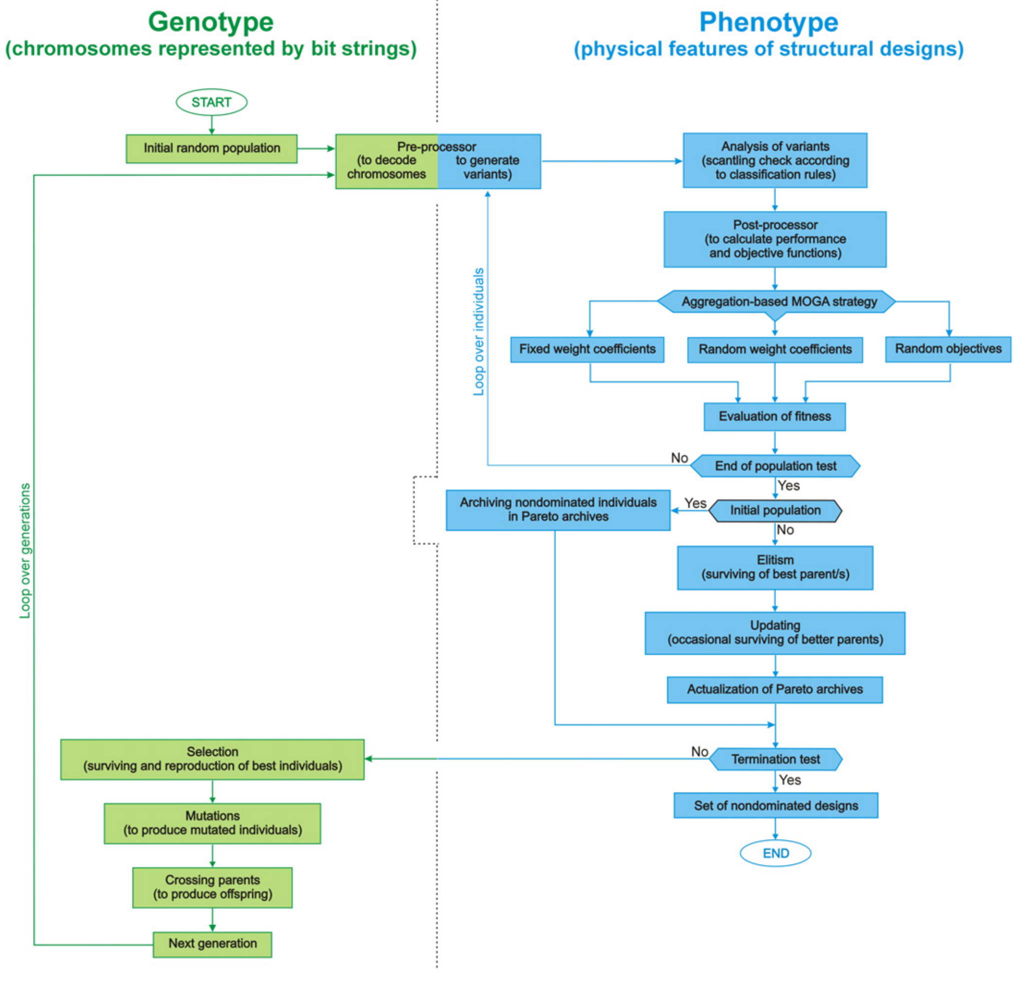

Fig. 7. Diagram presenting the main concepts of the developed computer code for the multi-objective optimization of seagoing ship structures, based on a genetic algorithm.

predecessor: (1) values of switches specifying the selection of strategy which is going to be used for the calculation of fitness function values basing on optimization criteria values, and (2) the values of weight coefficients assigned to fitness function components. The domination relation is checked over a set of individuals considered to be feasible. The set of non-dominated individuals, updated for each generation, constitutes an approximation of Pareto front. At the end, the value of fitness function, proportional to the individual selection probability and its participation in the genetic operations of the next generation, is calculated. Calculation of fitness function value takes into account the strategy of *MOGA (Multi-objective Genetic Algorithm)*, selected by the user. The computer software developed herein implements three MOGA selection strategies, Fig. 6: (1) strategy based on the scalarization of vector objective function, with appropriate weight coefficients, (2) strategy based on the scalar objective function with random weights of optimization criteria, (3) strategy executing the selection of variants on the basis of randomly selected single optimization criteria.

The code analyses the feasible individuals of successive generations, selects the non-dominated solutions and this way builds the archive of non-dominated solutions (Pareto-optimal ones). The archive of non-dominated variants is an external set and the variants which are recorded in it do not take part in the "breeding" of successive generation solutions, unless they have managed to survive in successive populations in a natural way, remaining not destroyed and not excluded by genetic operators. Process of building and evaluation of descendant populations of tested variants and the updating of non-dominated variants archive is repeated until a fixed number of generations used as a simulation halt condition is reached. After a simulation is completed, the non-dominated solutions preserved in the archive are recognized to be the sought solutions of a multi-objective optimization problem.

The developed code allows for the execution of one of aggregation-based multi-objective strategies used for the implementation of optimization criteria in the calculation of fitness function values: (1) fitness function is calculated using the values of weight coefficients w_s set by the user before the simulation run, which remain fixed during the whole run, (2) each time before the value of fitness function is calculated, all the weight coefficients w_s take independent random values, (3) each time before the value of fitness function is calculated, one randomly chosen weight coefficient w_s takes the value of one, while the values of remaining ones take the value of zero. It is then a method which main concept is based on the method of weighted objectives, belonging to the group of classical multi-objective optimization methods. The adopted evaluation criteria f_s are used as a linear combination with their weight coefficients w_s in equation (7), and this function is adopted as a main scalar optimization criterion F for the problem. The evaluation criteria f_s adopted for the multi-objective optimization problem become partial evaluation criteria of a scalar optimization criterion F of the entire problem. Strategies (1) and (2) are variants of a strategy based on a scalarization of objective vector function, having respectively deterministic or random values of weight coefficients w_s, while the strategy (3) is a strategy based on the selection of variants using a single, randomly chosen optimization criteria.

The calculation tool developed for use in optimization of ship structure should of course allow for accounting for a series of constraints imposed by design, local strength and overall strength. On the other hand, implementation of genetic algorithms requires that the equivalent problem is formulated without any constraints. Basing on the fact that genetic algorithms do not require continuity nor the existence of derivative functions, an external penalty function has been used Fox [12], Reklaitis et al. [33], Ryan [34], Vanderplaats [45]. The augmented objective function of unconstrained maximization problem $f(x)$, has been formulated as a penalty function:

$$f(x) = F(x) + \sum_{k=1}^{n_c} w_k P(x)_k^{r_k} \tag{7}$$

where: $F(x)$ – objective function in the constrained problem (eq. (6)), $P(x)_k$ – component of penalty function for the violation of k-th constraint, r_k – power exponent for k-th component of penalty function, w_k – penalty coefficient for the violation of k-th constraint, n_c – number of constraints.

As the augmented objective function $f(x)$ expressed by the relation Eq.(7) is: (1) defined, (2) single-valued, (3) ascending, having real values and positive in the search space, it has been adopted directly as the fitness function.

The values of parameters controlling the genetic operators are specified by the user before the start of simulation. Proper setting of these values is very important and requires extensive experience on the part of the user, but it is crucial for the attainment of the desired calculations convergence expressed in (1) quality of found solutions, (2) rapidity of their finding, and (3) required computation resources.

In case of multi-objective optimization problem we have to consider how to collect and present the information about the determined non-dominated solutions (Pareto-optimal) and how to archive them. It is commonly accepted to graphically present all the feasible solutions as points in the objective space. Only a part of them are going to be non-dominated (dominating, Pareto-optimal) solutions, and their set is going to be called a non-dominated set or a Pareto front containing trade-off solutions (actually we know this is going to be a set of non-dominated solutions approximating the Pareto set).

Non-dominated solutions are recorded in a separate set (file) which is continuously supplemented and updated during the simulation. The solutions collected in the non-dominated solutions set may be dealt within two possible ways: (1) set membership has no influence on the selection of individuals (egalitarian strategy) (2) individuals from the set (non-dominated solutions) enjoy a guaranteed participation in selection (elite strategy). Egalitarian strategy has been adopted in the underlying paper.

It is a well-known fact that in case many optimization criteria are used, it is going to be impossible to find a single best solution, as such a solution does not exist. In practice however the user awaits automatic or quasiautomatic determination of a single solution or a few solutions, which could be taken as a solution of the problem. Moreover, users are accustomed to the monitoring of evolution of a single value, which lets them evaluate the correctness of the calculation run, the convergence of solutions and the quality of solutions being found. In single-objective cases it is natural to monitor the values of fitness function and optimization criterion Sekulski [37]. In case of multi-objective optimization simultaneous evaluation of the evolving criteria is difficult to realization and interpretation. In order to alleviate this problem, the author has used a concept of ideal or utopia solution, well-known in literature Statnikov & Matosov [43], Cohon [4], Stadler [42], see Fig. 8a. In the generally accepted understanding an ideal point refers to the lowest values of all criteria analyzed singly and not together. In such a case it is then possible to locate a solution closest to the ideal point (nearest ideal solution), with the concept of closeness being understood here usually in the sense of Euclidean metrics.

The concept of non-dominated feasible solution nearest to the ideal objective is sufficient to find a single solution which may be considered to be “the best” solution of a multi-objective optimization problem. It is however inappropriate for the monitoring of evolution of non-dominated solution in the direction of theoretically lowest values of optimization criteria $f_i \to 0!$ as moving the set of non-dominated solutions in the desired direction may also take place with unchanging distances of these solutions from ideal solution, Fig. 9. The author has then introduced a concept of a asymptotic objective

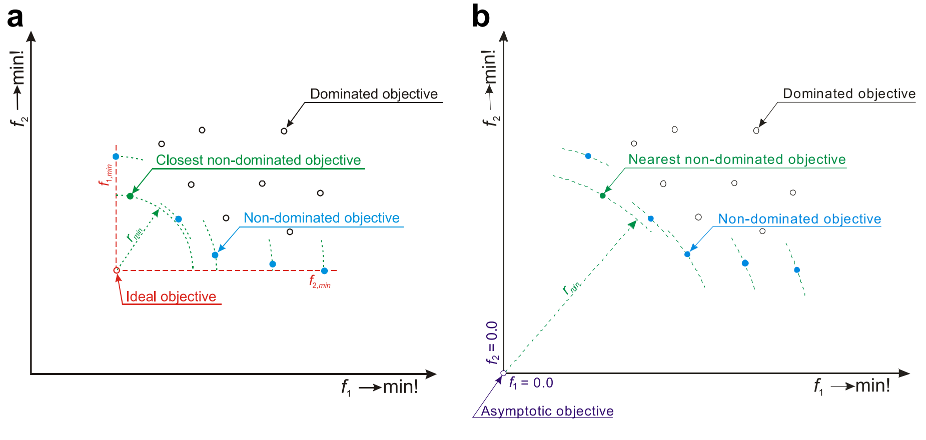

Fig. 8. (a) Graphical illustration of a non-dominated solution nearest to the ideal one, for the case of min! type criteria. (b) Graphical illustration of the concept of a non-dominated feasible solution nearest to a asymptotic objective, in case of min! type criteria.

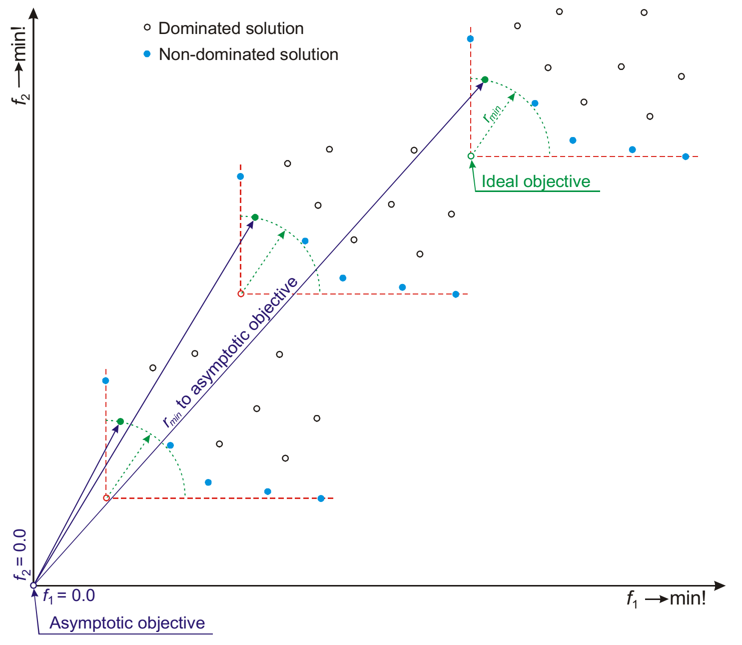

Fig. 9. Graphical illustration of the movement of solutions nearest to the ideal solution in the direction of asymptotic solution during the evolution, in case of min! type criteria; for constant distances from the ideal solution the distance from asymptotic one changes (evolves) instead.

(asymptotic solution), which represents an objective corresponding to asymptotical values of optimization criteria: $f_1 \rightarrow 0!$ $f_2 \rightarrow 0!$, Fig. 8b. Such a definition of a asymptotic solution it is possible to: (1) determine the distance between each non-dominated solution from and this point, and then choose a asymptotic closest solution, and additionally (2) monitor the evolution of set of non-dominated solutions distance from this solution during the simulation.

According to this definition, the location of ideal objective in the objective space is not fixed, and this means that its location changes during simulation, so the distance to such a moving objective is not a good indicator of solution quality. Therefore, a definition of asymptotic objective has been adopted in such a way as to make it stationary. It is then necessary to have such a definition which would make the co-ordinates of this objective constant in the objective space, which means fixed values of optimization criteria. It is then assumed that in case of min! type criteria the respective objectives (co-ordinates) are equal to zero. However, in case of max! type criteria the user shall set the value of these objectives (co-ordinates) as some known from experience values of respective criteria which are impossible to attain, but which are going to be approached asymptotically by the non-dominated solutions.

4. Multi-objective optimization model of seagoing ship hull structure

The effectiveness of the developed evolutionary algorithm for the multi-objective optimization of seagoing ship structures has been checked by solving the multi-objective optimization problem for the structure of a spatial midship section of a passenger-car catamaran ferry design Auto Express 82 m, Fig. 10. In the earlier work Sekulski [37] the models developed for the single-objective optimization have been presented and discussed: (1) ship structure model, (2) optimization model and (3) genetic model. These models have been properly adapted by extending them for the needs of multi-objective

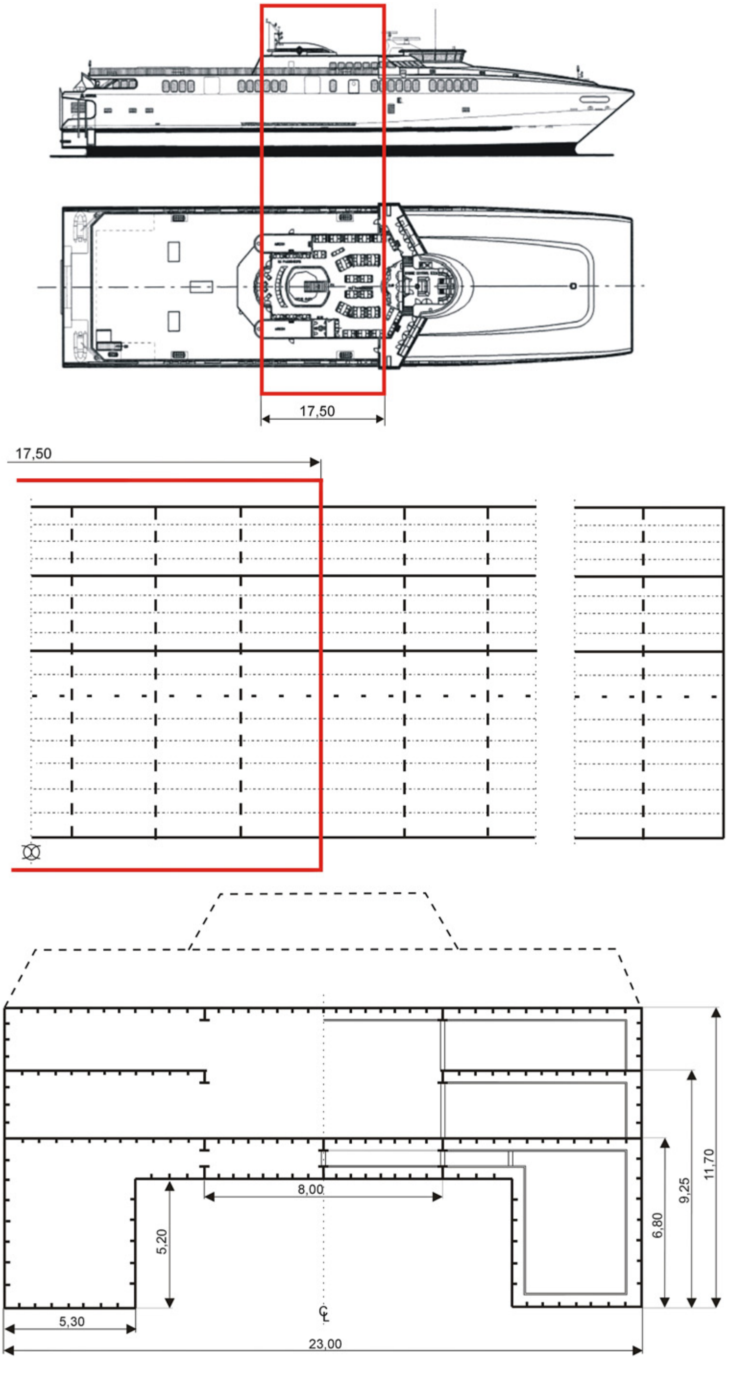

Fig. 10. The diagram of a model of a spatial section of a fast passenger-car ferry Auto Express 82 m, used for the research.

Table 1
Control (steering) parameters of research simulations.

No.	Designation sym*i*	Specification (n_g, n_i, n_p, p_m, p_c, *c_strategy*, *n_x_site_min*, *n_x_site_max*, p_u, *elitist*, *w_strategy*, w_1, w_2)
1.1.	sym1-1	(10.000, 5.000, 10, 0.086, 0.800, 1, 1, 7, 0.33, *yes*, 2, 0.5, 0.5)
1.2.	sym1-2	(10.000, 5.000, 10, 0.086, 0.800, 1, 1, 7, 0.33, *yes*, 4, random in [0,1], random in [0,1])
1.3.	sym1-3	(10.000, 5.000, 10, 0.086, 0.800, 1, 1, 7, 0.33, *yes*, 3, random 0 or 1, random 0 or 1)

optimization. The model of structure has been reused without changes. The optimization model has been extended by defining a scalar objective function of a following form:

$$F(x) = F(f_1(x), f_2(x)) = w_1 f_1(x) + w_2 f_2(x) \tag{8}$$

where f_1 is structural weight of midship block-section taken to optimization, f_2 is an area of the outer surface of structural members subjected to cleaning and painting operations (surface area for maintenance) in the section, w_1 and w_2 are weight coefficients used for partial optimization criteria. As a consequence the following form of fitness function has been adopted:

$$f(x) = F(x) + \sum_{k=1}^{n_c} w_k P(x)_k^{r_k} = w_1 f_1(x) + w_2 f_2(x) \sum_{k=1}^{n_c} w_k P(x)_k^{r_k} \tag{9}$$

where all the symbols are as outlined before.

Three groups of constraints have been assumed in the paper: (1) behaviour constraints to prevent the structure from taking the state considered as destroyed or loss of functional properties, (2) side constraints determining the scope of design variables, (3) geometrical constraints imposing limitations on the dimensions between the design variables were introduced due to the “good practice” rules. The constraints were discussed in details in the former paper Sekulski [37].

The behaviour constraints assumed in the paper are in principle strength constraints and enable calculation of plate thicknesses and section moduli of stiffeners and web frames in accordance to the classification rules UNITAS [49]. It was assumed that individual regions of the ship hull are subject to one of the following loading: (1) pressure of water dependant on the speed and the navigation region, (2) weight of cars, or (3) weight of equipment and passengers. Values of pressure were calculated according to the classification rules.

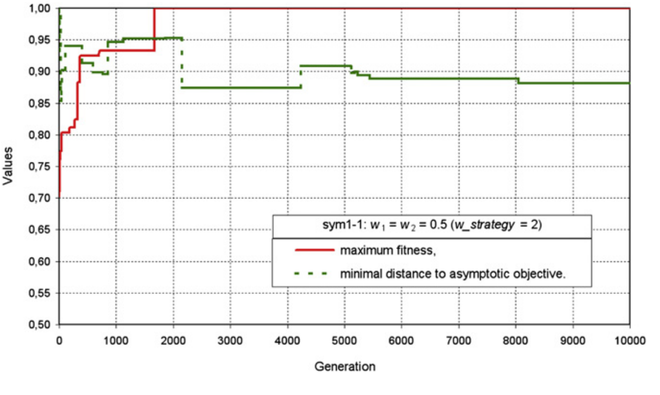

Fig. 11. The results of multi-objective genetic optimization of ship structure with respect to structure weight f_1 and the area of element surface f_2 in case of fixed values of optimization criteria weight coefficients $w_1 = w_2 = 0.5$, (sym1-1); the curves present the evolution of a highest value of fitness function $f_{\max}$, the lowest value of non-dominated solution distance from a asymptotic one; the values are dimensionless and standardized in [0,1] range in relation to the highest values found during the simulation.

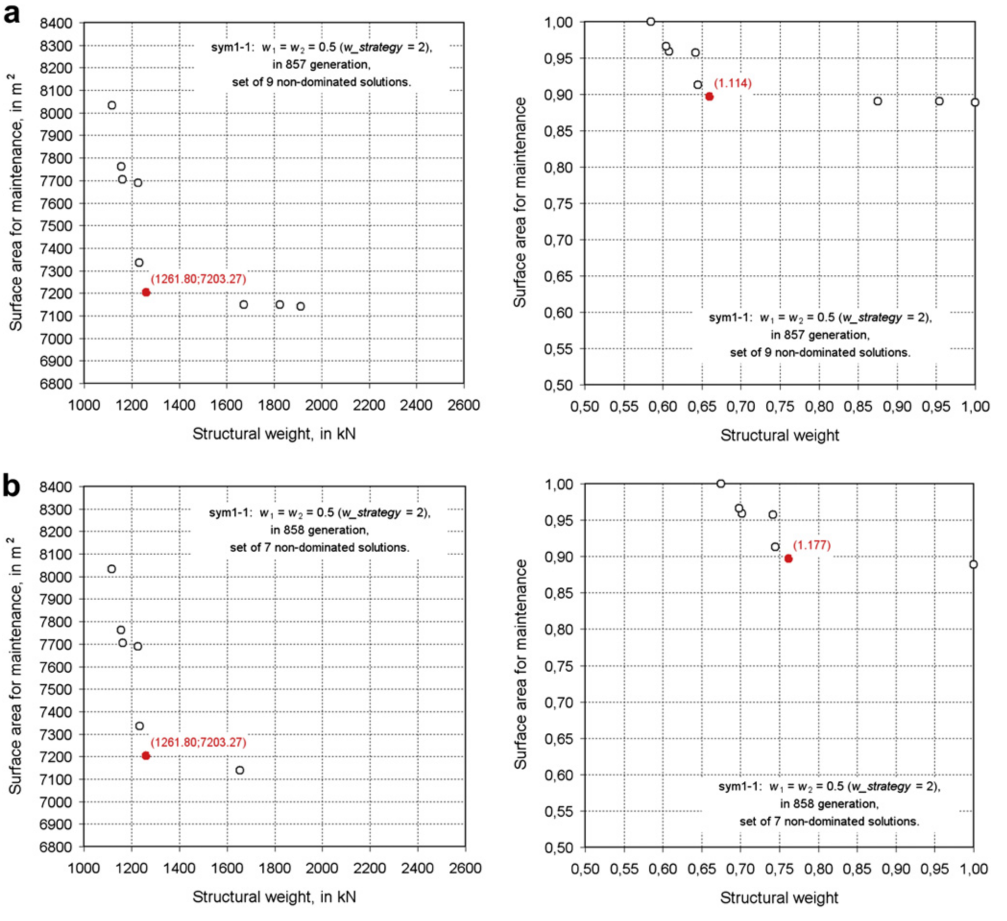

Fig. 12. Change of a structure of non-dominated solutions set when moving from generation 857 (a) to generation 858 (b); during the genetic multi-objective optimization of ship structure with respect to structure weight f_1 and surface area f_2 in case of fixed values of optimization criteria weight coefficients $w_1 = w_2 = 0.5$, (sym1-1); circles represent non-dominated solutions, red points represent non-dominated solutions closest to the ideal one; dimensionless values are normalized to the interval [0,1] in relation to the highest values in the set; change of non-dominated solution set structure and change of non-dimensional (normalized) of distance of nearest solution from asymptotic solution caused not by change of values of partial optimization criteria but only change of set of non-dominated solutions set structure can be observed. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article).

As has already been stated in section 3, three aggregation-based multi-objective evolutionary strategies for taking account of the partial optimization criteria $f_1(x)$ and $f_2(x)$ used in the scalar objective function (e.g. 8), and therefore also in the fitness function (eq. 9): selection of variants using the scalar objective function (eq. 8) with the values of weight coefficients w_1 and w_2 set by the user ($w\_strategy = 2$), selection of variants using the scalar objective function (eq. 8) with the values of weight coefficients w_1 and w_2 randomly generated in the range [0, 1] ($w\_strategy = 4$), selection of variants using the randomly selected single partial optimization criteria $F(x) = w_1 f_1(x)$ or $F(x) = w_2 f_2(x)$ ($w\_strategy = 3$) which is implemented by a random selection of a single nonzero weight criterion.

The genetic model takes into account a scalarized fitness function in the form (9). In this case the set of genetic model parameters set for each simulation run sym*i* includes 13 elements:

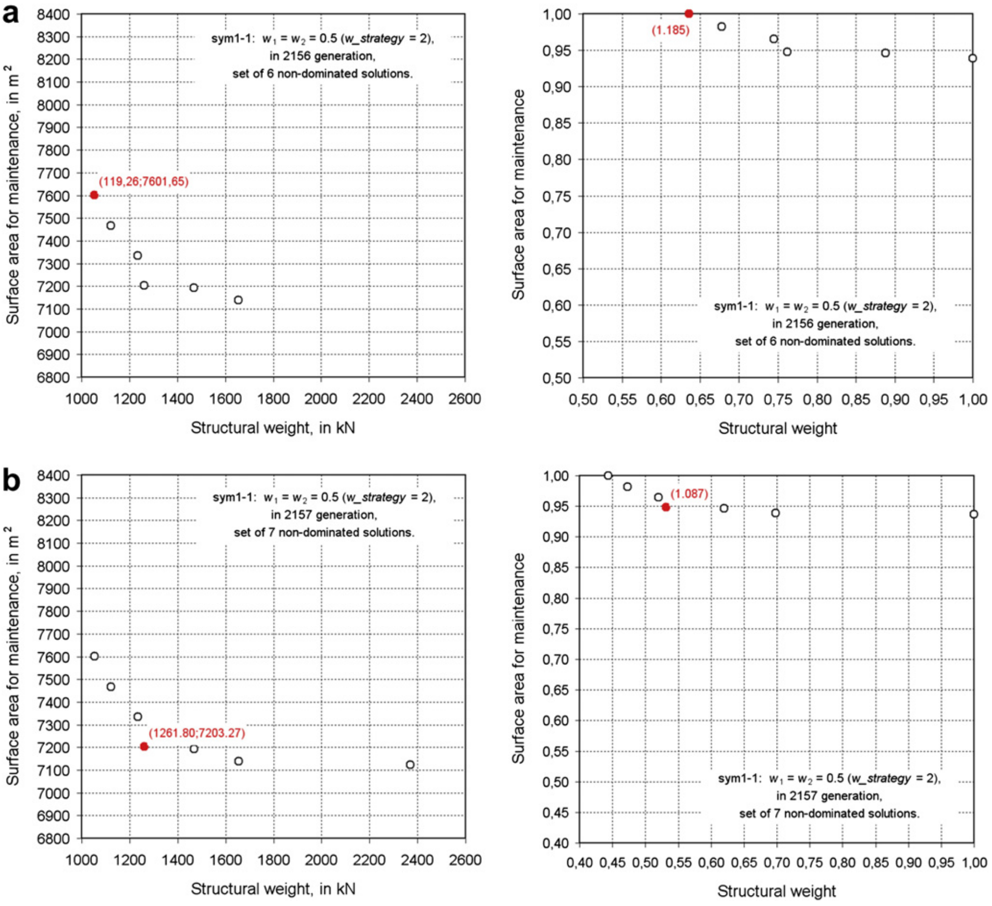

Fig. 13. Change of a structure of non-dominated solutions set when moving from generation 2156 (a) to generation 2157 (b); during the genetic multi-objective optimization of ship structure with respect to structure weight f_1 and surface area f_2 in case of fixed values of optimization criteria weight coefficients $w_1 = w_2 = 0.5$, (sym1-1); circles represent non-dominated solutions, red points represent non-dominated solutions closest to the asymptotic one; dimensionless values are normalized to the interval [0,1] in relation to the highest values in the set; change of non-dominated solution set structure and change of non-dimensional (normalized) of distance of nearest solution from asymptotic solution caused not by change of values of partial optimization criteria but only change of set of non-dominated solutions set structure can be observed. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article).

$$symi = \left(n_g,\ n_i,\ n_p,\ p_m,\ p_c,\ c\_strategy,\ n\_x\_site\_\min,\ n\_x\_site\_\max,\ p_u,\ elitist,\ w\_strategy,\ w_1,\ w_2\right)$$

where n_g-number of generations, n_i-size of population, n_p-number of pretenders, p_m-mutation probability, p_c-crossover probability, *c_strategy*-denotation of crossover strategy (0 for fixed, 1 for random number of crossover points), *n_x_site_min*-the lowest number of crossover points, *n_x_site_max*-the greatest number of crossover points, p_u-update probability, *elitist*-logical variable to switch on ($elitism = yes$) and off ($elitism = no$) the pretender selection strategy, *w_strategy*-denotation of strategy for scalarization of objective function, w_1-weight coefficient for weight of structure, w_2-weight coefficient of surface area of structural element for cleaning and painting. These 13 parameters control the successive simulation runs and identify them uniquely for the adopted structure model.

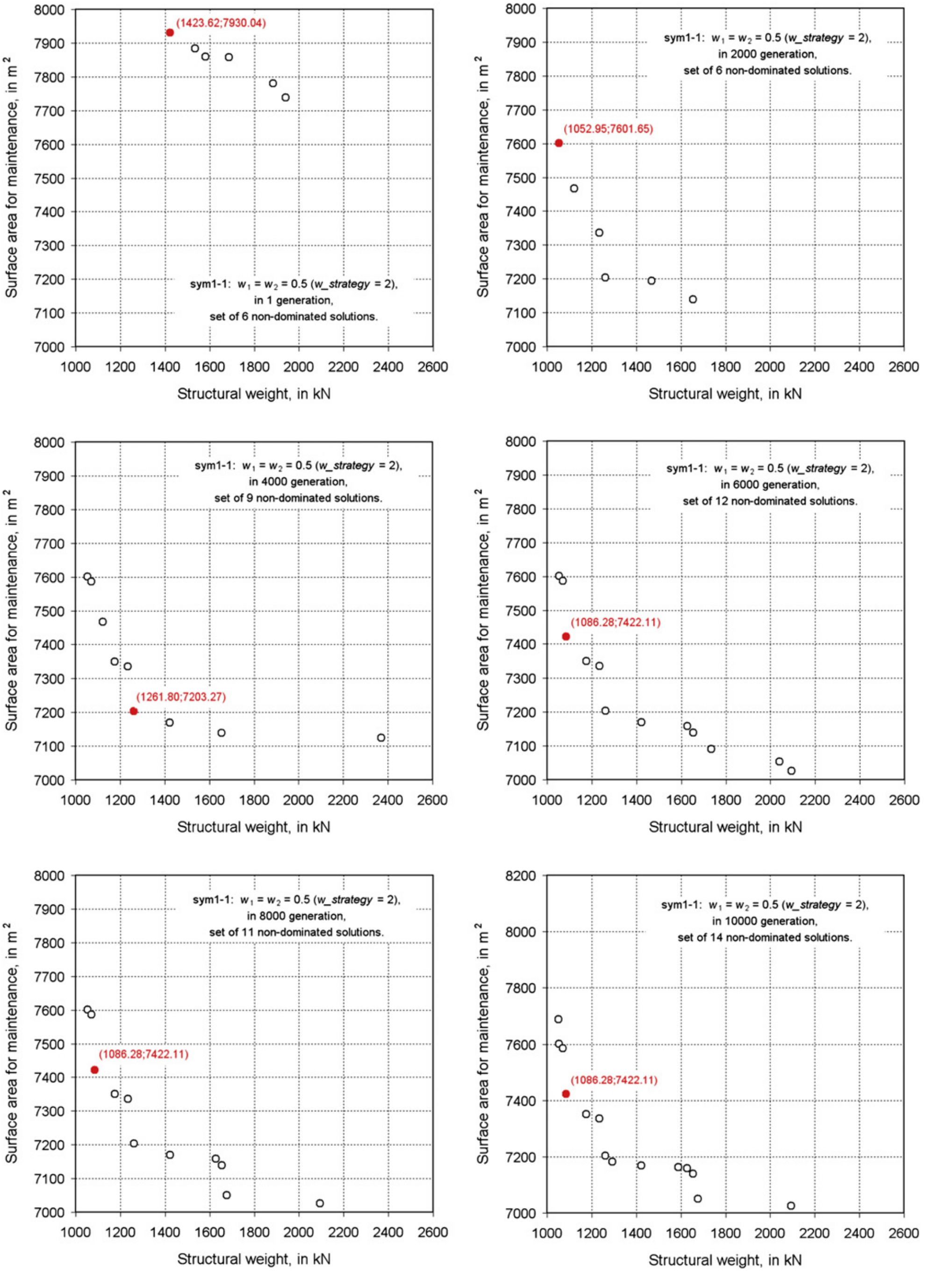

Fig. 14. Evolution of non-dominated solutions set during the genetic multi-objective optimization of ship structure with respect to structure weight f_1 and surface area f_2 in case of fixed values of optimization criteria weight coefficients $w_1 = w_2 = 0.5$, (sym1-1); black circles represent non-dominated solutions, red dots represent non-dominated solutions closest to the asymptotic one. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article).

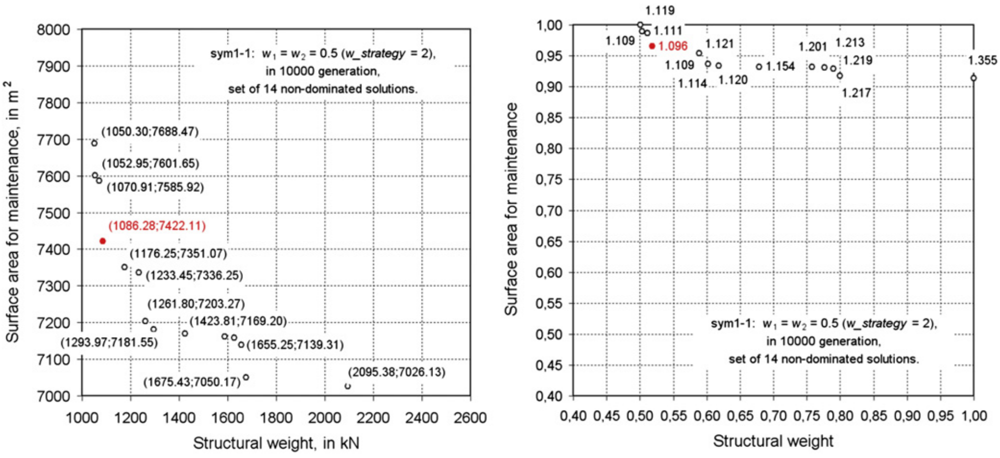

Fig. 15. Detailed specification of non-dominated solutions set obtained during the genetic multi-objective optimization of ship structure with respect to structure weight f_1 and surface area f_2 in case of fixed values of optimization criteria weight coefficients $w_1 = w_2 = 0.5$, (sym1-1); black circles represent non-dominated solutions, red dots represent non-dominated solutions closest to the asymptotic one; dimensionless values are normalized to the interval [0,1] in relation to the highest values in the set.[3] (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article).

5. Research calculations–the search for set of non-dominated solutions

5.1. Research calculations program

In order to verify the suitability of the proposed method and the computer code developed for the seeking of Pareto-optimal solutions of the formulated multi-objective seagoing ship structure optimization problem, a number of calculation experiments have been carried out, Table 1, using the ship structure models earlier formulated and discussed in chapter 4.

From multi-objective optimization point of view, aim of simulation was search for non-dominated variants with respect to two optimization criteria with varying strategies for setting the values of weight coefficients for various criteria: (1) in the simulation marked as sym1-1 fixed values of weight coefficients are used for whole simulation: $w_1 = 0.5$ and $w_2 = 0.5$; which refers to a classical method of weighted criteria, (2) in the simulation marked as sym1-2 the values of weight coefficients w_1 and w_2 have been generated by software as random variables in the range [0, 1], which was done independently for each variant whenever the value of fitness function is calculated, (3) in the simulation marked as sym1-3 the values of weight coefficients w_1 and w_2 have been generated by software as random variables equaling 0 or 1, which was done independently for each variant whenever the value of fitness function is calculated; the value of 1 was used only for one, randomly selected criterion, with the remaining ones equaling 0. In all simulations the functions of penalties imposed for the violations of constraints are active.

The research calculations were carried out first of all for two-objective problems, because in such a case there is a possibility to present the obtained results graphically in a multitude of ways, which enables their interpretation and analysis.

[3] Normalization of the optimization objective values makes it possible to calculate the distance from asymptotic objective in the Euclidean sense in cases when the axes of the co-ordinate system represent the objectives denoted in various units.

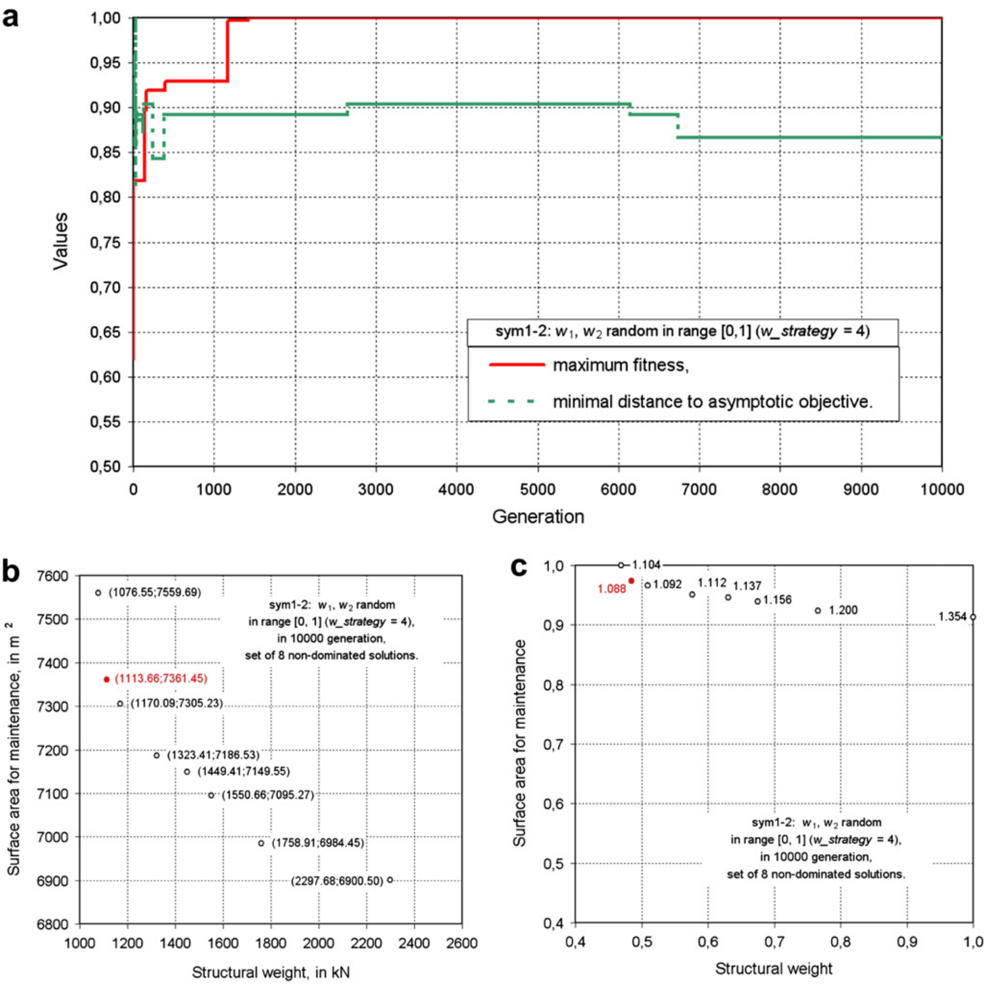

Fig. 16. Results of genetic multi-objective optimization of ship structure with respect to structure weight f_1 and surface area f_2 in case of random values of optimization criteria weight coefficients w_1 and w_2 in range of [0, 1], (sym1-2); a) the curves present the evolution of a highest value of fitness function f_{max} , the lowest value of non-dominated solution distance from a asymptotic one, b) detailed specification of final non-dominated solutions set in objective space, c) detailed specification of final non-dominated solutions set in normalized objective space; black circles represent non-dominated solutions, red circles represent non-dominated solution closest to the asymptotic one; dimensionless values are normalized to the interval [0,1] in relation to the highest values in the set. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article).

5.2. Results of research calculations

The results of simulation sym1-1 are going to be discussed in a most detailed way, presenting the results characteristic for a proposed method. As far as the remaining simulations are concerned, only the most important results are going to be presented.

The Fig. 11 presents the evolution of macroscopic values characterizing the evolution of generated and evaluated ship structure solutions population in simulation sym1-1: (1) greatest fitness function value f_{max}, (2) lowest distance the feasible solution to the asymptotic solution. Multi-objective optimization of ship structure with regard to structure weight f_1 and the surface area for maintenance f_2 in case of fixed values of weight coefficients $w_1 = w_2 = 0.5$ used for optimization criteria. The figure shows a desired continuous rise of a greatest value of fitness function f_{max} indicating rising quality of the best

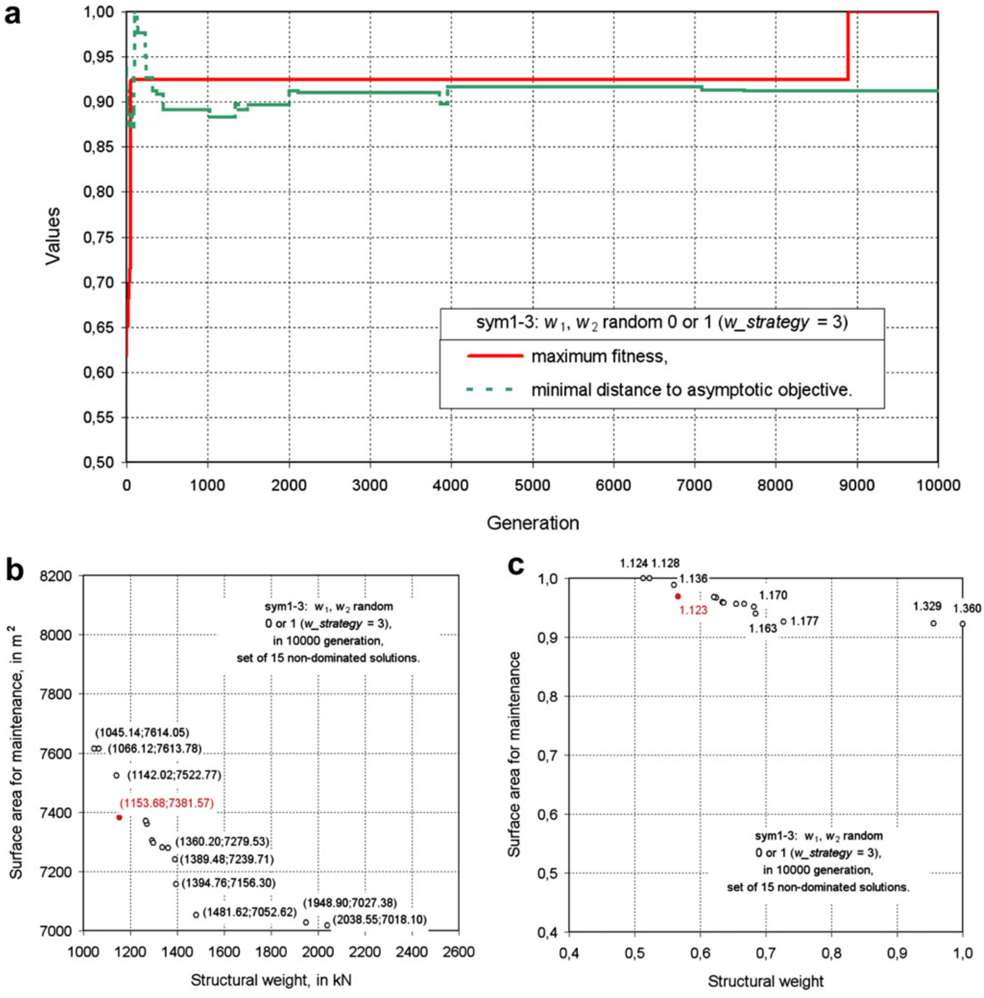

Fig. 17. Results of genetic multi-objective optimization of ship structure with respect to structural weight f_1 and surface area f_2 in case of random values of weight coefficients w_1 and w_2 equaling 0 or 1, (sym1-3); a) the curves present the evolution of a highest value of fitness function f_{max} , the lowest value of non-dominated solution distance from a asymptotic one, b) detailed specification of final non-dominated solutions set in objective space, c) detailed specification of final non-dominated solutions set in normalized objective space; black circles represent non-dominated solutions, red dots represent non-dominated solution closest to the asymptotic one; dimensionless values are normalized to the interval [0,1] in relation to the highest values in the set. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article).

generated test solutions. Highest values of fitness function saturate already in 1668 generation, which means that in the following generations no solutions were generated which would be better adapted in the sense of the fitness function used, and the computation resources were squandered replicating the non-dominated solutions set. The lowest distance between a non-dominated solution and the asymptotic solution changes during the evolution, but above the threshold of 87.38% of the highest value found during the simulation this takes place in a miniscule extent. For example, in the 857th generation the distance of the closest solution to the asymptotic solution is 1.114, see Fig. 12a, while in the successive 858th generation the distance of the closest solution to the asymptotic solution increased to 1.177, Fig. 12b. For the same solution the change of the distance to the asymptotic solution occurs only due to the change of the structure of the set of non-dominated solutions. On the other hand, for example, in the

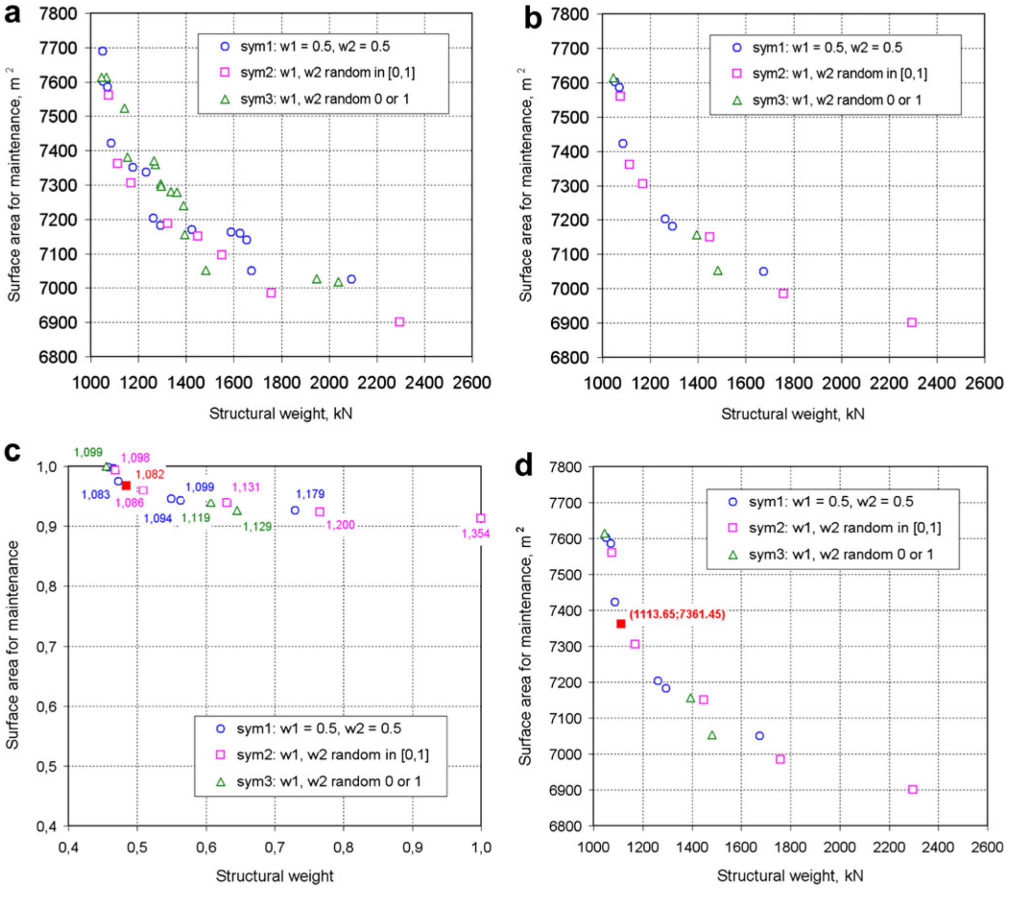

Fig. 18. Selection of single, recommended solution of multi-objective optimization problem using non-dominated solution sets obtained in three series of computer simulations: (a) temporary set composed of non-dominated solutions of each simulation, (b) selection of non-dominated solutions in temporary set, (c) determination of distance of non-dominated solutions to asymptotic objective in normalized objective space, (d) values of optimization criteria for found closest solution $f_1(x)$ = 1113.65 kN and $f_2(x)$= 7361.45 m^2.

2156th generation the distance of the closest solution to the asymptotic solution is 1.187, Fig. 13a, while in the successive 2157th generation the distance of the closest solution to the asymptotic solution decreased to 1.087, Fig. 13b. In this case the change of the set structure caused another solution, already present in the set, became the solution closest to the asymptotic solution.

In the Fig. 14 the evolution of the structure of non-dominated solutions set is shown using as examples the selected time-based cross-sections of this set, e.g. for 1, 2000, 4000, 6000, 8000 and 10.000 generations. A systematic growth of the population of non-dominated solutions set is apparent with 6, 6, 9, 12, 11 and 14 non-dominated solutions respectively in the consecutive time-based cross-sections, as well as the desired evolution of this set in the direction of more advantageous values of partial optimization criteria.

Fig. 15 presents a detailed structure of a non-dominated solutions set of a last generation, presented in a physical space of objectives and the normalized space of objectives. In can be seen that a set of non-dominated solutions including 14 variants of ship structure has been found during the simulation. For each non-dominated variant the values of optimization criteria have been specified as: $f_1(x)$-structural weight and $f_2(x)$-cleaned/painted surface area. The designer may select for further development one of

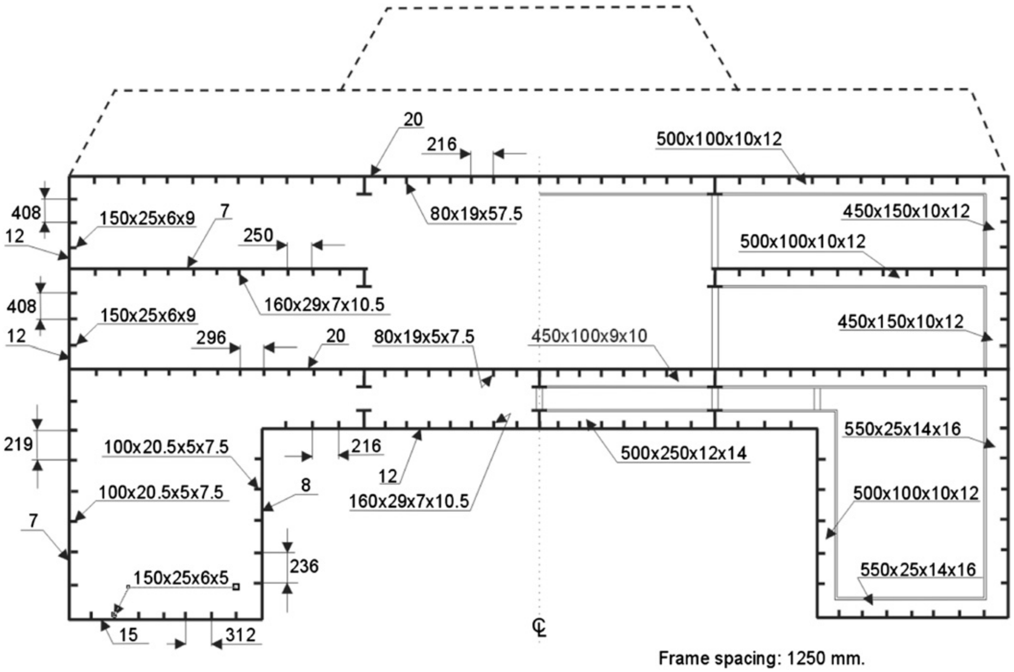

Fig. 19. Dimensions and scantlings of vessel structure recommended as the result of multi-objective optimization; structural material: 5083-H111 aluminium alloys for plates, 6082-T6 aluminium alloys for profile extrusions.

these variants or a group of them deemed by him/her to be the best. For the variant closest to the asymptotic solution, which was found in 5116 generation, the distance equaling 1.096 in the normalized objective space, the structural weight is $f_1(x) = 1086.28$ kN and the cleaned/painted surface area is $f_2(x) = 7422.10$ m^2. This variant may be recommended if there is a need to select a single solution for the formulated ship structure multi-objective optimization problem.

The Fig. 16a presents the evolution of macroscopic values characterizing the evolution of generated and evaluated ship structure variants population in simulation sym1-2 in case of random values of weight coefficients w_1 and w_2 in range of $[0, 1]$. The figure shows a desired continuous rise of a greatest value of fitness function f_{max} indicating rising quality of the best generated test variants. Highest values of fitness function saturate already in 1416 generation. The lowest distance between a non-dominated solution and the asymptotic solution changes during the evolution, but above the threshold of 86,69% of the highest value found during the simulation this takes place in a miniscule extent.

Fig. 16b and c presents a detailed structure of a non-dominated solutions set of a last generation. In can be seen that a set of non-dominated solutions including 8 variants of ship structure has been found during the simulation. For the solution closest to the asymptotic solution, which was found in 6145 generation, the distance equaling 1.088 in the normalized objective space, the structure weight is $f_1(x) = 1113.66$ kN and the surface area for maintenance is $f_2(x) = 7361.45$ m^2.

The Fig. 17a presents the evolution of macroscopic values characterizing the evolution of generated and evaluated ship structure variants population in simulation sym1-3 in case of random values of weight coefficients w_1 and w_2 equaling 0 or 1 used for optimization criteria. The figure shows a desired continuous rise of a greatest value of fitness function f_{max} indicating rising quality of the best generated test solutions. Highest values of fitness function saturate not before 8888 generation in means very close to the end of simulation. The lowest distance between a non-dominated solution and the asymptotic solution changes during the evolution, but above the

Table 2
Values of design variables recommended as the result of multi-objective optimization.

No.	Symbol	Description	Value
1	2	3	4
1	x_1	serial No. of mezzanine deck plate	5
2	x_2	serial No. of mezzanine deck bulb	6
3	x_3	serial No. of mezzanine deck T-bulb	48
4	x_4	number of web frames	13
5	x_5	number of mezzanine deck stiffeners	29
6	x_6	serial No. of superstructure I plate	8
7	x_7	serial No. of superstructure I bulb	5
8	x_8	serial No. of superstructure I T-bulb	47
9	x_9	number of superstructure I stiffeners	5
10	x_{10}	serial No. of inner side plate	6
11	x_{11}	serial No. of inner side bulb	2
12	x_{12}	serial No. of inner side T-bulb	48
13	x_{13}	number of inner side stiffeners	21
14	x_{14}	serial No. of bottom plate	9
15	x_{15}	serial No. of bottom bulb	5
16	x_{16}	serial No. of bottom T-bulb	52
17	x_{17}	number of bottom stiffeners	16
18	x_{18}	serial No. of outer side plate	5
19	x_{19}	serial No. of outer side bulb	2
20	x_{20}	serial No. of outer side T-bulb	52
21	x_{21}	number of outer side stiffeners	30
22	x_{22}	serial No. of wet deck plate	8
23	x_{23}	serial No. of wet deck bulb	6
24	x_{24}	serial No. of wet deck T-bulb	51
25	x_{25}	number of wet deck stiffeners	36
26	x_{26}	serial No. of main deck plate	10
27	x_{27}	serial No. of main deck bulb	1
28	x_{28}	serial No. of main deck T-bulb	46
29	x_{29}	number of main deck stiffeners	26
30	x_{30}	serial No. of superstructure II plate	8
31	x_{31}	serial No. of superstructure II bulb	5
32	x_{32}	serial No. of superstructure II T-bulb	47
33	x_{33}	number of superstructure II stiffeners	5
34	x_{34}	serial No. of upper deck plate	10
35	x_{35}	serial No. of upper deck bulb	1
36	x_{36}	serial No. of upper deck T-bulb	48
37	x_{37}	number of upper deck stiffeners	36

threshold of 88.30% of the highest value found during the simulation this takes place in a miniscule extent.

Fig. 17b and c present a detailed structure of a non-dominated solutions set of the last generation. In can be seen that the set of non-dominated solutions including 15 ship structural variants has been found during the simulation. For the solution closest to the asymptotic solution which was found in 7611 generation, the distance of 1.123 in the normalized objective space, the structural weight is $f_1(x) = 1153.68$ kN and the surface area for maintenance is $f_2(x) = 7381.57$ m^2.

5.3. Analysis of results and the conclusions obtained from the research simulations

Three series of the computer simulations confirmed effectiveness of the developed computational algorithm and computer code for solution of the formulated unified topology-size ship structure multi-objective optimization problem. In a result of calculations an approximation of the Pareto-optimal set containing in each simulation from a few to more than ten non-dominated solutions was found. Obtained results do not allow for unequivocal decision on the advantage of either of the examined objective function aggregation strategies was best, yet visual assessment of the shape of the obtained of approximations of the Pareto-optimal set suggest an advantage of the strategy with random values of weight coefficients w_s (sym1-2) and the least effectiveness of the strategy with fixed values of the weight coefficients w_s (sym1-1). Effectiveness of the strategy with random selection of single optimization criteria in the selection process (sym1-3) is intermediate. In the case of the constrained problems it also turns that the components of the penalty functions introduce a random contribution to the fitness function thus causing the strategy with fixed weight coefficients w_s to be practically a strategy similar to the two others and it also allows to find the approximation of the Pareto-optimal set with the adequate accuracy.

From the found sets of the compromise solutions a user can in the next stage select one or a few solutions applying additional premises which are not included in the optimization model. He can also select suggested non-dominated solutions the closest to the asymptotic solutions: sym1-1: $f_1(x) = 1086.28$ kN, $f_2(x) = 7422.10$ m^2, sym1-2: $f_1(x) = 1113.65$ kN, $f_2(x) = 7361.45$ m^2, sym1-3: $f_1(x) = 1153.68$ kN, $f_2(x) = 7381.57$ m^2.

Since in this way three solutions are obtained, the next question is which of them can be recommended as the best.[4] I this place the author suggests the following procedure. Non-dominated solution sets obtained in subsequent simulations can be merged into a temporary solution set presented in Fig. 18a. In this set only a part of solutions is non-dominated solutions, Fig. 18b. In the set of 15 non-dominated solutions created using the results of three simulations a distance of each of them to the asymptotic objective in normalized objective space can be determined, Fig. 18c. The least distance, equal to 1.082, was obtained for solution $f_1(x) = 1113.65$ kN and $f_2(x) = 7361.45$ m^2 found in simulation sym1-2 (random values of weight coefficients w_1 and w_2 in range [0, 1]). The solution can be recommended as a single solution of the formulated problem of multi-objective optimization.

The recommended non-dominated solution was obtained for the values of design variables specified in Table 2. Corresponding dimensions of the ship cross-section are given in Fig. 19.

6. Summary and conclusions

A developed tool for the solution of the unified problem of the multi-objective optimization of topology and scantlings of the seagoing ship hull dimensions with the accuracy typical for the initial design. A computational example developed using the high-speed passenger-car twin-hull ferry, design Auto Express 82, was presented and discussed in details, Strength criteria were taken from the adopted classification rules.

It should be remembered that the developed multi-objective optimization algorithm is based on the random processes therefore the obtained results should be interpreted in the statistical sense. It means that the simulations and obtained results may be not representative. Slight change of the developed models or control parameters may result in a different simulation course and obtaining different results.

Further systematic investigations of effectiveness of the proposed strategies including repeated computations different only by the evolution history for statistic confirmation of effectiveness of the strategies are necessary.

The problem has been investigated for almost 20 years with the intention to develop effective evolutionary algorithms of multi-objective optimization. The investigations resulted in developing a number of advanced algorithms employing Pareto-domination relation. Methods employing scalarization of the objective function were found too simplified and not effective. In the latest years the results of investigations were published, however, suggesting that the evolutionary algorithms based on the domination relation can be ineffective in solving problems described by a model containing more than three optimization criteria. Some researchers indicate that in such cases the scalarization algorithms, earlier abandoned, can be proven effective. The algorithms were investigated in the present paper for ship structures.

[4] Let us remember that in the multi-objective optimization there is not the single best solution of the problem and the formulated recommendation should be treated as a subjective choice by a person taking decision.

The scalarization method is effective even in the case of the fixed values of the weight coefficients since in the case of the constrained problem the components of the penalty function introduce a random influence to the fitness function. The method is thus closer to the method employing the random weight coefficients of the optimization criteria.

Despite relatively simple computational model of the structure and the lack of direct methods for strength assessment, evolutionary solution of a multi-objective, unified problem of simultaneous optimization of topology and scantlings calls for significant computational effort. Searching ways to shorten the computational time is therefore justified. One of the methods can be, for example, a decomposition of the problem and parallel solution of the partial optimization problems.

References

[1] Abraham A, Jain L, Goldberg R. Evolutionary multiobjective optimization. Springer; 2005.
[2] Binh TT, Korn U. MOBES: A multiobjective evolution strategy for constrained optimization problems. In: The third international conference on genetic algorithms (Mendel 97), Brno, Czech Republic; 1997; 176–182.
[3] Coello CAC, Lamont GB, Veldhuizen DA. Evolutionary algorithms for solving multi-objective problems. Springer; 2007.
[4] Cohon JL. Multiobjective programming and planning. New York: Academic Press; 1978.
[5] Das PK. Reliability – Based Design Procedure of Stiffened Cylinder Using Multiple Criteria Optimisation Techniques. In: Proceedings, OTC 1993, Vol. 3, No. 7236, p. 297–313; 1993.
[6] Das PK, Tolikas C, Morandi AC, Zanic V, Warren NF. Multiple criteria synthesis technique applied to the reliabilty based structural design of hull components of a fast swath ship. In: FAST '93 Proceedings, Japan, Tokyo, vol. 1, pp. 473–487, 1993.
[7] Davis L. Handbook of genetic algorithms. New York: Van Nostrand; 1991.
[8] Deb K. Multi-Objective optimization using evolutionary algorithms. John Wiley & Sons; 2001.
[9] Deb K, Agrawal S, Pratab A, Meyarivan T. A fast elitist non-dominated Sorting genetic algorithm for multi-objective optimization: NSGA-II. KanGAL Report 200001. Kanpur, India: Indian Institute of Technology; 2000.
[10] Eschenauer H, Koski J, Osyczka A. Multicriteria design Optimisation. Berlin: Springer-Verlag; 1990.
[11] Fonseca CM, Fleming PJ. Genetic Algorithms for Multiobjective Optimization: Formulation, Discussion and Generalization. In: 5th International Conference on Genetic Algorithms, Proceedings; 1993; 416–423.
[12] Fox RL. Optimization methods for engineering design. Reading: Addison-Wesley Publishing Company, Inc.; 1971.
[13] Goldberg DE. Genetic algorithms in search, optimization and machine learning. Addison-Wesley Pub.; 1989.
[14] Hajela P, Lin CY. Genetic search strategies in multicriterion optimal design. Structural Optimization 1992;4:99–107.
[15] Horn J, Nafpliotis N, Goldberg DE. A Niched Pareto genetic algorithm for multiobjective optimization. In: First IEEE Conference on evolutionary computation, 1. IEEE World Congress on Computational Intelligence; 1994. p. 82–7.
[16] Hughes EJ. Multiple single objective Sampling. In: Proc. of 2003 Congress on evolutionary computation. 2003, 2678–2684.
[17] Hughes EJ. Evolutionary many-objective optimization: many once or one many? In: Proc. of 2005 Congress on evolutionary computation. 2005, 222–227.
[18] Hutchinson KW, Sen P, Buxton IL, Hills W. Multiple Criteria Design Optimisation of Ro-RO Passenger Ferries with Consideration of recently proposed Probabilistic Stability Standards. In: Proceedings 7th International Symposium on Practical Design of Ships and Mobile Units, 1. The Netherlands: PRADS '98, The Hauge; 1998. p. 303–12.
[19] Ishibuchi H, Murata T. Multi-objective genetic local search algorithm. In: Proceedings of IEEE International Conference on evolutionary computation (ICEC'96). Piscataway, NJ: IEEE Press; 1996. p. 119–24.
[20] Jang CD, Shin SH. A study on the optimal structural design for oil tankers using multi objective optimization. In: Proceedings, 1. Int. Marine Design Conference IMDC; 1997. p. 217–31.
[21] Jaszkiewicz A. On the computational efficiency of multiple objective metaheuristics: the knapsack problem case study. European Journal of Operational Research 2004;158:418–33.
[22] Jianguo W, Zuoshui X. Symmetric solution of fuzzy multi-objective optimization for ship structure, 10. Joun. East China Shipbuilding Institute; 1996 (1): 1-7.
[23] Knowles J, Corne D. In: The Pareto archived evolution strategy: a new baseline algorithm for multiobjective optimisation. Washington, D.C: 1999 Congress on Evolutionary Computation; 1999. p. 98–105.
[24] Kursawe F. A variant of evolution strategies for vector optimization. In: Schwefel H-P, Manner R, editors. Parallel problem solving from Nature. Berlin: Springer; 1991. p. 193–7.
[25] Michalewicz Z. Genetic algorithms + data structures=evolution Programs. Berlin-Heidelberg: Springer-Verlag; 1996.
[26] Murata T, Ishibuchi H. MOGA: multi-objective genetic algorithms. In: Proceedings of the second IEEE international conference on evolutionary computation; 1995. p. 289–94.
[27] Okada T, Neki I. Utilization of genetic algorithm for optimizing the design of ship hull structure, 171. Japan: J.S.N.A.; 1992. 71–83.
[28] Osyczka A. Evolutionary algorithms for single and multicriteria design optimization. Heidelberg: Physica-Verlag; 2002.
[29] Pareto V. Cours D'Economie Politique, vol 1. Lausanne: F. Rouge; 1896.
[30] Parsons MG, Singer D. A fuzzy logic agent for design team communications and negotiations. In: COMPIT 2000. Berlin: Conf. Postdam; 2000.
[31] Purshouse RC, Fleming PJ. Evolutionary many-objective optimization: an exploratory analysis. In: Proc. of 2003 Congress on evolutionary computation; 2003. p. 2066–73.
[32] Ray T, Sha OP. Multicriteria optimisation model for a containership design. Marine Technology 1994;31(4):258–68.
[33] Reklaitis GV, Ravindran A, Ragsdell KM. Engineering optimization. Methods and applications. New York: John Wiley and Sons; 1983.

[34] Ryan DM. Penalty and barrier functions. In: Gill PE, Murray W, editors. Numerical methods for constrained optimization. London: Academy Press; 1974.
[35] Sarker R, Mohammadian M, Yao X, editors. Evolutionary optimization, part III, multi-objective optimization. Boston: Kluwer Academic Publishers; 2002.
[36] Schaffer JD. Multiple objective optimization with vector evaluated genetic algorithms. In: Proceedings of an international conference on genetic algorithms and their applications; 1985. p. 93–100.
[37] Sekulski Z. Least-weight topology and size optimization of high speed vehicle-passenger catamaran structure by genetic algorithm. Marine Structures 2009;22(4):691–711.
[38] Sen P, Yang JB. An investigation into the influence of preference modelling in ship design with multiple objectives. In: Proceedings, PRADS '95, vol. 2. Society of Naval Arch. of Korea; 1995. 1252–1263.
[39] Sen P, Yang JB. Multiple criteria decision support in engineering. London: Springer-Verlag; 1998.
[40] Shi WB. In-service assessment of ship structures: effect of general corrosion on ultimate strength. In: Spring Meteting. RINA; 1992.
[41] Srinivas N, Deb K. Multiobjective optimization using nondominated sorting in genetic algorithms. Evolutionary Computation 1995;2(3):221–48.
[42] Stadler W. Multiobjective optimization in engineering and in the sciences. New York: Plenum Press; 1988.
[43] Statnikov RB, Matosov JB. Multicriteria optimization and engineering. New York: Chapman&Hall; 1995.
[44] Trincas G, Zanic V, Grubisic I. Comprehensive concept design of fast RO-RO ships by multi-atribute decision-making. In: Proceedings, IMDC '94, Delft, 1994, p. 403–417.
[45] Vanderplaats GN. Numerical optimization techniques for engineering designs. New York: McGraw-Hill; 1984.
[46] Veldhuizen DA. (1999) Multiobjective evolutionary algorithms: classifications, analyses, and new innovations. Ph. D. Thesis, Air Force Institute of Technology, Wright-Patterson AFB, Ohio.
[47] Zitzler E, Laumanns M, Thiele L. SPEA-2: improving the strength Pareto evolutionary algorithm. Evolutionary Methods for Design. In: Optimization and control with applications to industrial problems, Greece; 2001; 95–100.
[48] Zitzler E, Thiele L. Multiobjective optimization using evolutionary algorithms-A comparative case study. In: Parallel problem solving from nature–PPSN, Amsterdam; 1998; 292–301.
[49] UNITAS, Rules for the Construction and Classification of High Speed Craft, 1995.