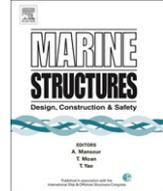




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# Least-weight topology and size optimization of high speed vehicle-passenger catamaran structure by genetic algorithm

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### ABSTRACT

Selection of the “best” or “optimum” engineering design has always been a major concern of designers. Reduction of hull weight is the most important aim in the structural design of many ship types. But the ability of designers to produce optimal designs of ship structures is severely limited by the calculation techniques available for this task. Complete definition of the optimal structural design requires formulation of size–topology–shape–material optimization task unifying optimization problems from four areas and effective solution of the problem. So far a significant progress towards solution of this problem has not been achieved. In other hand in recent years attempts have been made to apply genetic algorithm (GA) optimization techniques to design of ship structures. An objective of the paper was to create a computer code and investigate a possibility of simultaneous optimization of both topology and scantlings of structural elements of large spatial sections of ships using GA. In the paper GA is applied to solve the problem of weight minimization of a high speed vehicle-passenger catamaran structure with several design variables as dimensions of the plate thickness, longitudinal stiffeners and transverse frames and spacing between longitudinals and transversal members. Results of numerical experiments obtained using the code are presented. They show that GA can be an efficient optimization tool for simultaneous design of topology and sizing high speed craft structures.

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## 1. Introduction

In extreme case a ship can be designed using a paper sheet and a calculator. It requires uncomplicated mathematical calculations and selection of plate thicknesses and types of profiles from the variety of products available according to the production standards not involving any optimization process. Such “manual” realization of the whole process of the structural design is labour-consuming, long-lasting and obtained solutions, in most cases, cannot be considered as optimal. Such a situation is unsatisfactory and cannot be presently accepted.

Computer-aided design and application of the numerical algorithms of analysis and optimization allow to speed up the calculations, decrease the labour consumption and support designer's intuition and experience in the analysis of the intermediate solutions, especially when the designed structure is not typical or the decision is to be taken quickly and, as it frequently happens, the data set is incomplete, e.g. during the pre-contractual negotiations. For these reasons very important is that many various optimization algorithms have been developed for a defined class of problems.

Structural design should be commenced very early in the process of designing ships soon after defining basic operational requirements, e.g. main particulars, speed, range, capacity since it has essential meaning for fulfilling tasks of the unit at acceptable level. Errors at this stage cannot be usually satisfactorily compensated at further stages of designing – if not removed they cause the greatest losses. Simultaneously correct and quick decisions result in the greatest advantages and influence the overall success of the enterprise in a decisive manner. A primary objective of the ship structural optimization is to find the optimum positions of structural elements, also referred to as topological optimization, shapes (shape optimization) and scantlings (sizing optimization) of structural elements for an objective function subject to constraints [27]. Formally, selection of structural material can also be treated as a part of the optimization process (material optimization). An essential task of the ship structure optimization is to reduce the structural weight, therefore most frequently the minimum weight is assumed as an objective function. Topological optimization means searching for optimal existence and space localization of structural elements while shape optimization is searching for optimal shape of ship hull body. Sizing optimization can also be expressed as a process of finding optimum scantlings of structural elements with fixed topology and shape. Selection of the structural material is usually not an explicit optimization task but is rather done according to experience and capability of a shipyard. Application of the optimization methods when selecting material usually consists of obtaining a few of independent solutions for given values of variables describing mechanical properties of material. Systematic optimization procedures for the selection of structural material are applied directly in rare cases. Optimization of structure of laminates is an example of such an optimization problem.

Shape optimization problems are solved within computational fluid dynamics. Advanced methods of CFD combined with robust random optimization algorithms allowed for optimizing a ship hull shape. Practical application of results is usually very difficult due to problems related to building ship hulls with optimal shapes (e.g. too slender hull shape to accommodate propulsion systems) as well as insufficient ship capacity. Despite continuous growth of computer capabilities and efficiency of optimization methods, progress in optimization of structural topology is very slow: only small-scale optimization problems were examined [2,27]. First optimization procedures for solution of sizing optimization problems such as SUMT allowed for searching optimal scantlings of structural elements using analytical methods for stress evaluation [19,22]. This approach offered quick optimization process but the disadvantage was that the algorithm had to be adjusted to each specific structure. Employing FEM it was possible to develop general computational tools [15,38], yet the time necessary for stress evaluation became significantly longer. To avoid this difficulty two approaches were suggested; developing more efficient mathematical algorithms of search [9], or dividing the optimization problem into two levels [16,18,26–28,30,31], so called Rational Design.

Thus the process of ship structural design and optimization can be considered in the four following areas: optimization of shape, material, topology and scantling. Due to complexity of optimization problem related to ship structures, only partial optimization tasks are formulated in each of the four areas independently. No significant attempt to unify the optimization problems has been done so far.

Problems of ship structural design contain many design variables of values having large range. It means that the set of variants in a given search space is numerous. In such cases application of review

methods is ineffective in terms of time and impossible for acceptance in practice. Simultaneously, basic criteria and limitations are derived from the strength analysis and usually are nonlinear with respect to design variables. Nonlinear form of function dependencies makes difficulty in practice application of the differential calculus. It is thus necessary to find an alternative solution.

Preliminary developments proved the genetic algorithm (GA) could be an efficient tool for ship structural optimization [23–25,29,37]. The GA is proposed as a method for improving ship structures through more efficient exploration of the search space. The results of research on the GA application for optimization of high speed craft hull structure topology and sizing optimization are presented in the paper while the optimization of shape and material was not covered. The main ideas of GA are briefly described in Section 2. The computer code for structural optimization by GA is described in Section 3. Structural, optimization and genetic models of a simplified fast craft hull structure are described in Sections 4–6, respectively. The results of application of the computer code to the optimal design of the analysed structure are given in Section 7. Some general conclusions are formulated in Section 8.

## 2. Genetic algorithm

The genetic algorithm belongs to the class of evolutionary algorithms that use techniques inspired by the Darwinian evolutionary theory such as inheritance, mutation, natural selection, and recombination (or crossover) [3,10,20,21].

The genetic algorithm is typically implemented in the form of computer simulations where a population of abstract representations (called chromosomes) of candidate solutions (called individuals) to an optimization problem evolves gradually towards better solutions. Traditionally, solutions are represented in the binary system as strings of 0s and 1s but different encodings are also possible. The evolution starts from a population of completely random individuals and is continued in subsequent generations. In each generation, the fitness of the whole population is evaluated, multiple individuals are stochastically selected from the current population (based on their fitness), modified (mutated or recombined) to form a new population which becomes current in the next generation. Procedures of creation and evaluation of the successive generations of trial solutions are repeated until the condition of termination of computations is fulfilled, e.g. forming a predefined number of generations or lack of correction of the fitness function in a number of successive generations. The best variant found is then taken as the solution of the optimization problem.

A powerful stochastic search and optimization computational technique controlled by evolutionary principles can be effectively used to find approximate solutions of combinatorial optimization problems. They can be easily applied to optimization problems with discrete design variables which are typical in ship structural optimization. GA uses nondeterministic scheme and is not associated with differentiability or convexity. This is why using GA the global optimum can be reached in the search space more easily than by traditional optimization techniques. Another useful advantage is that it is very easy to use the discrete serial numbers of rolled or extruded elements (it means plates and bulbs) and number of structural elements in each region of ship hull as design variables because, by nature, the GA uses discrete design variables (design variables in the form of floating point numbers are also possible). However, there are some difficulties in optimization processes with the use of GA due to the trouble of converging to the actual optimum. Employing GA user should accept the fact that he will never know how close to the global optimum the search was terminated. He can only expect that the best final variants will be concentrated in the vicinity of local extrema and, possibly, global extremum. The final solution, believed to be optimal, is only an approximation of the global optimum. Level of this approximation cannot be estimated as the precise methods of examination of convergence of the GA were not developed so far. It is necessary to investigate robustness and convergence before application of GA to the structural optimization.

## 3. Computer code for genetic optimization of structures

Applicability of GA for solution of the optimization problems unifying topology and scantling optimization of ship structure was verified using computer simulation. A computer code was developed adding the modules of the pre-processing, scantling analysis and post-processing to the genetic

modules (selection, mutation, crossover) which form the Simply Genetic Algorithm (SGA). The flowchart of the code is shown in Fig. 1.

In the computer code the optimization problem is solved creating random populations of trial solutions. All principal operators of the basic evolutionary process [5,10,21] are used in the code: natural selection, mutation and crossover. Two additional operators: the elitist [6] and update operator [34] – are introduced for the selection as well. The genetic operators used in the computer code are described in details in Section 6.4.

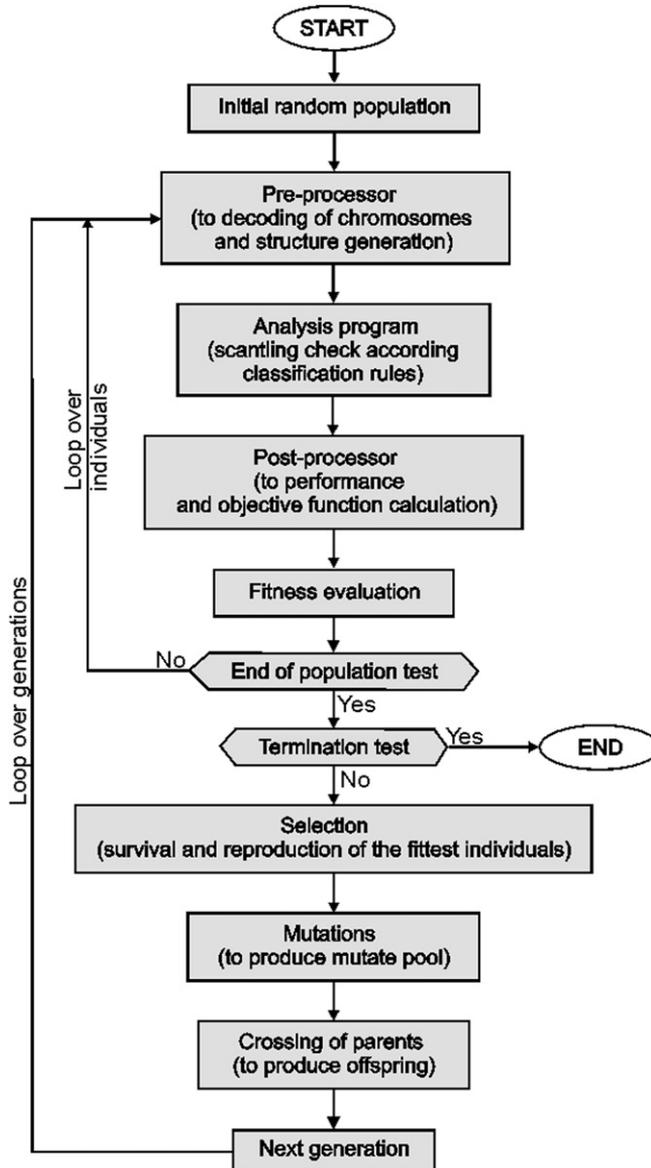


Fig. 1. Flowchart of computer code for ship structure optimization by genetic algorithm.

In the computer code a population of individuals of a fixed size is randomly generated. Each individual is characterized by a string of bits and represents one possible solution to the ship structure topologies and sizing. Each new created variant of solution (an individual being a candidate to the progeny generation) is analysed by the pre-processor. In the pre-processor binary strings of chromosomes (genotypes) are decoded into the corresponding strings of decimal values representing design variables (phenotypes). Then for the actual values of the design variables defining spatial layout of the structural elements (topology) and their scantlings it is checked whether the actual configuration complies with the rules of the classification society. In the next step performance of solution is evaluated and it is checked whether the variant meets the constraints. At the end the value of the objective function is calculated for each variant – weight of the structure, and the value of the fitness function which is used for ordering the variants necessary to starting of selection. Variants are ordered with respect to this value. Knowing adaptation of each variant the random process is restarted to select variants of the successive progeny generation.

After selection the code determines randomly which genes of these whole population will mutate. This population is then mutated where small random changes are made to the mutants to maintain diversity. After that the mutate pool is created. Then decision is made how much information is swapped between the different population members. The mutated individuals are then paired up randomly and mated in the process commonly known as crossover. The idea is to derive better qualities from the parents to have even better offspring qualities. That is done by creating, with fixed probability, “cutting points” and then the parts of the chromosomes located between “cuts” are interchanged. The mating process is continued until the full population is generated. The resulting population member is then referred to as an offspring. The newly generated individuals are then re-evaluated and given fitness score, and the process is repeated until it is stopped after a fixed number of generations. The best strings (individuals) found can be used as near-optimal solutions to the optimization problem.

All genetic parameters are specified by the user before the calculations. The population size, number of design variables and number of bits per variable, the total genome length, number of individuals in the population are limited by the available computer memory.

#### 4. Structural model of ship hull

A model based on the Austal Auto Express 82 design developed by Austal [13,32] was applied for the optimization study. The general arrangement of the Austal Auto Express 82 vessel is shown in Fig. 2. Main particulars of the ship are given in Fig. 3. The vessel and his corresponding cross and longitudinal sections are shown in Fig. 4. For seagoing ships the application domain of initial stage design is clearly the cylindrical and prismatic zone of ship's central part. For this reason the analysis of a midship block-section ( $17.5 \times 23.0 \times 11.7$  m) was taken. Bulkheads form boundaries of the block in the longitudinal direction. In the block 9 structural regions can be distinguished. The transverse bulkheads were disregarded to minimize the number of design variables.

All regions are longitudinally stiffened with stiffeners; their spacing being different in each structural region. The transverse web frame spacing is common for all the regions. Both types of spacing, stiffener and transverse frame, are considered as design variables.

The structural material is aluminium alloy having properties given in Table 1. The 5083-H111 aluminium alloys are used for plate elements while 6082-T6 aluminium alloys are used for bulb extrusions. The plate thicknesses and the bulb and T-bulb extruded stiffener sections are assumed according to the commercial standards and given in Tables 2–4. Bulb extrusions are used as longitudinal stiffeners while T-bulb extrusions are used as web frame profiles. Practically, the web frames are produced welding the elements cut out of metal sheets. Dimensions of the prefabricated T-bar elements are described by four design variables (web height and thickness, and flange breadth and thickness). In the case of extruded bulb a single variable is sufficient to identify the profile, its dimensions and geometric properties. It delimits the computational problem and accelerates analysis. The strength criteria for calculation of plate thicknesses and section moduli of stiffeners and web frames are taken in accordance to the classification rules [36]. It was assumed that bottom, wet deck, outer side and superstructures I and II are subject to pressure of water dependant on the speed and the navigation region. The main deck was loaded by the weight of the trucks transmitted through the tires,

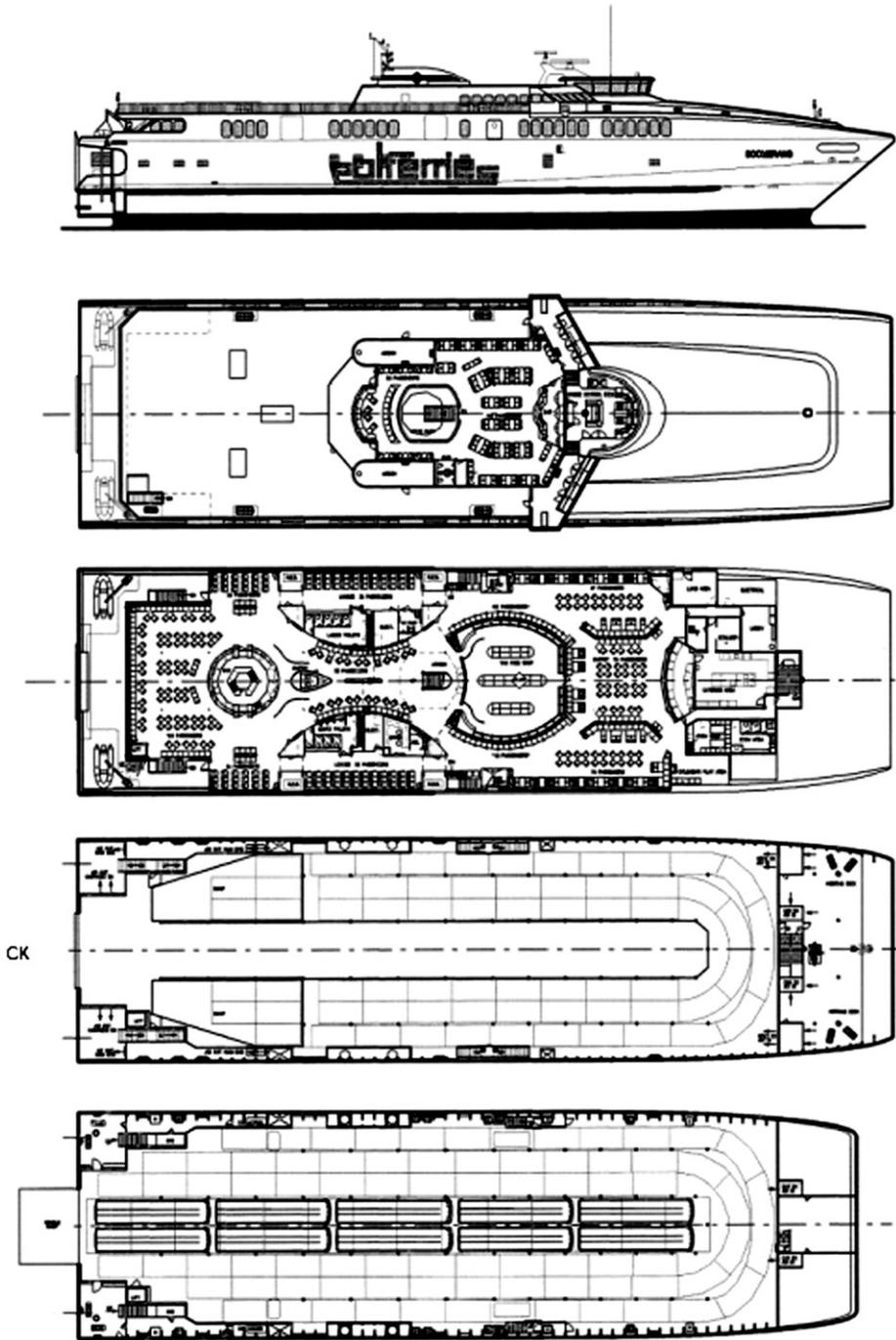


Fig. 2. High speed vehicle-passenger catamaran, type Austal Auto Express 82 – general arrangement [32].

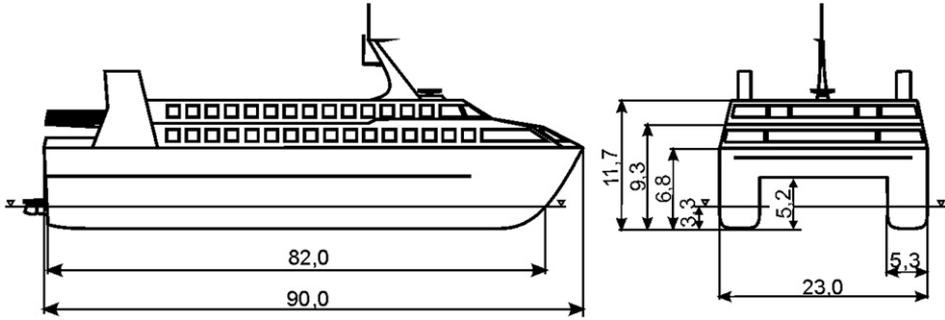


Fig. 3. High speed vehicle-passenger catamaran, type Auto Express 82 – main particulars.

mezzanine deck – weight of the cars, while the upper deck – the weight of equipment and passengers. Values of pressure were calculated according to the classification rules.

All genetic parameters are specified by the user before the calculations. This option is very important; the control of the parameter permits to perform search in the direction expected by the designer and in some cases it allows much faster finding solution. The population size, number of variables and number of bits per variable, the total genome length, number of individuals in the population are limited by the available computer memory.

A minimum structural weight (volume of structure) was taken as a criterion in the study and was introduced in the definition of the objective function and constraints defined on the base of classification rules. Where structural weight is chosen as the objective function its value depends only on the geometrical properties of the structure (if structural material is fixed). The assumed optimization task is rather simple but the main objective of the study was building and testing the computer code and proving its application for structural topology and sizing optimization of a ship hull.

### 5. Formulation of optimization model

In the most general formulation to solve a ship structural optimization problem means to find a combination of values of the vector of design variables  $\mathbf{x} = \text{col}\{x_1, \dots, x_i, \dots, x_n\}$  defining the structure which optimizes the objective function  $f(\mathbf{x})$ . The design variables should also meet complex set of constraints imposed on their values. The constraints formulate the set of admissible solutions. It is assumed that all functions of the optimization problem are real and a number of constraints are finite. Considering computational costs an additional requirement may also be formulated that they should be as small as possible.

As the minimum value of function  $f$  is simultaneously the maximum value of  $-f$ , therefore the general mathematical formulation of the both optimization problems reads:

find vector of design variables:  $\mathbf{x} = \text{col}\{x_1, \dots, x_i, \dots, x_n\}$ ;  $x_{i,\min} \leq x_i \leq x_{i,\max}$ ,  $i = 1, \dots, n$

$$\begin{aligned} & \text{minimize (maximize)} && f(\mathbf{x}) \\ & \text{subject} && h_k(\mathbf{x}) = 0, k = 1, 2, \dots, m' \\ & && g_j(\mathbf{x}) \geq 0, j = m' + 1, m' + 2, \dots, m \end{aligned} \tag{1}$$

where  $\mathbf{x}$  is a vector of  $n$  design variables,  $f(\mathbf{x})$  is objective function,  $h_k(\mathbf{x})$  and  $g_j(\mathbf{x})$  are constraints.

In the present formulation a set of 37 design variables is applied, cf. Table 5 and Fig. 5. Introduction of design variable representing the number of transversal frames in a considered section:  $x_4$ , and numbers of longitudinal stiffeners in the regions:  $x_5, x_9, x_{13}, x_{17}, x_{21}, x_{25}, x_{29}, x_{33}, x_{37}$  enables simultaneous optimization of both topology and scantlings within the presented unified topology-scantling optimization model.

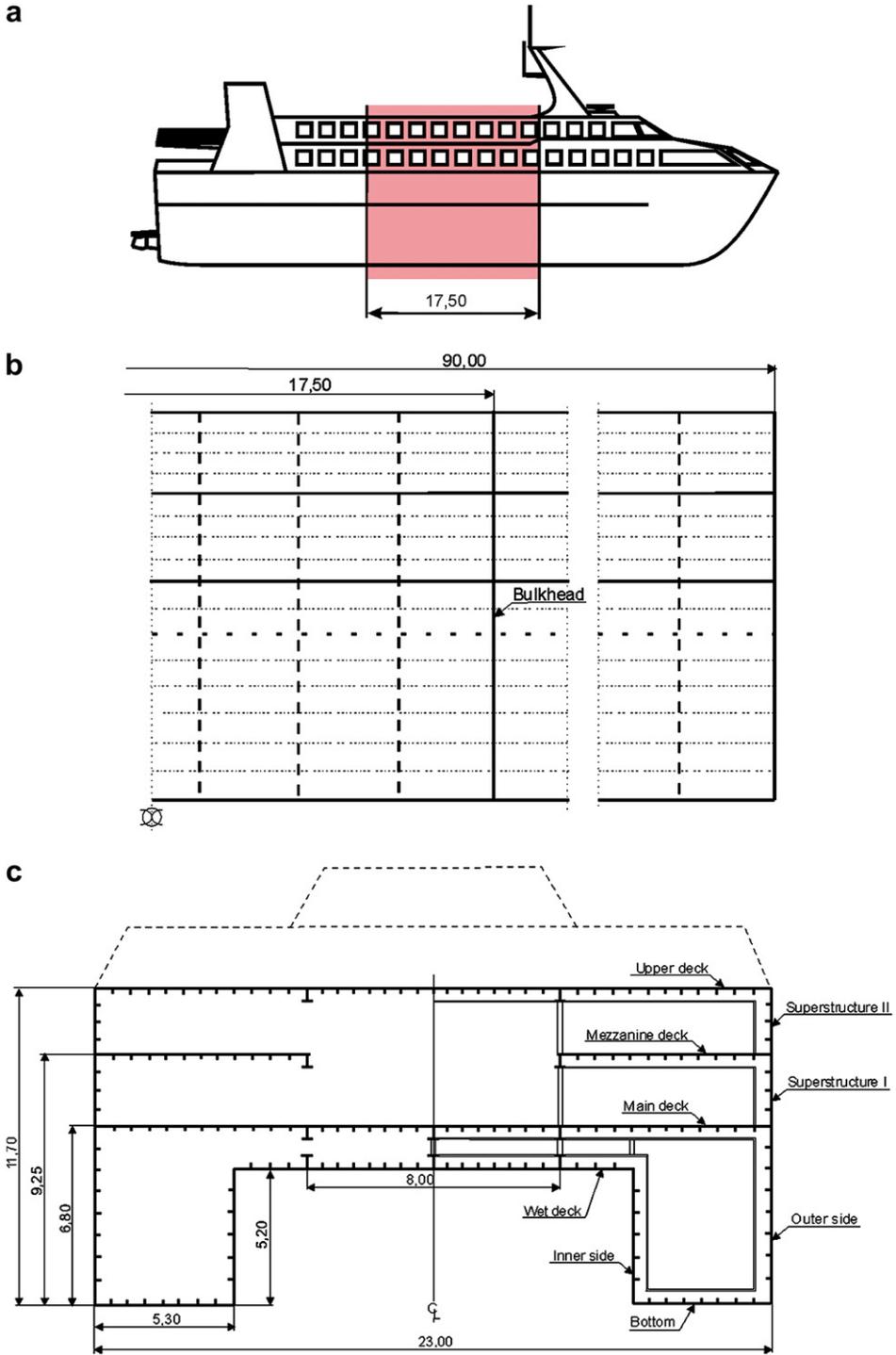


Fig. 4. Assumed model of craft – midship block-section, frame system and structural regions.

**Table 1**

Properties of structural material – aluminium alloys.

No.	Property	Value
1	Yield stress $R_{0.2}$	125 (for 5083-H111 alloy) N/mm <sup>2</sup> 250 (for 6082-T6 alloy) N/mm <sup>2</sup>
2	Young modulus $E$	70,000 N/mm <sup>2</sup>
3	Poisson ratio $\nu$	0.33
4	Density $\rho$	26.1 kN/m <sup>3</sup>

Numbers of stiffeners and transverse web frames, varying throughout the process of optimization, determine corresponding spacings of them. Scantlings and weights of structural elements: plating, stiffeners and frames are directly dependant on the stiffeners and frames spacings – topological properties of the structure.

Optimizing the structural topology of the ship, a difficult dilemma is to be solved concerning a relation between the number of structural elements in longitudinal and transverse directions and their dimensions, influencing the structural weight. Constraints should also be considered related to the manufacturing process and functional requirements of the ship, e.g. transportation corridors, supporting container seats on the containerships (typically by longitudinal girders and floors in the double bottom) or positioning supports on the girders in the distance enabling entry of cars on ro-ro vessels.

Objective function  $f(\mathbf{x})$  for optimization of the hull structure weight is written in the following form:

$$f(\mathbf{x}) = \sum_j^r w_j SW_j, \quad r = 9 \quad (2)$$

where  $r$  – number of structural regions;  $SW_j$  – structural weight of the  $j$ -th structural region; and  $w_j$  – relative weight coefficient (relative importance of structural weight) of regions varying in the range [0,1].

The behaviour constraints, ensuring that the designed structure is on the safe side, were formulated for each region according to the classification rules [36] constitute a part of the set of inequality constraints  $g_j(\mathbf{x})$ :

– required plate thickness  $t_{j,\text{rule}}(\mathbf{x})$  based on the permissible bending stress:

$$t_j - t_{j,\text{rule}}(\mathbf{x}) \geq 0 \quad (3)$$

where  $t_j$  – actual value of plate thickness in  $j$ -th region,

– required section moduli of stiffeners  $Z_{s,j,\text{rule}}(\mathbf{x})$ :

$$Z_{s,j} - Z_{s,j,\text{rule}}(\mathbf{x}) \geq 0 \quad (4)$$

where  $Z_{s,j}$  – actual value of the section modulus of stiffeners in  $j$ -th region,

– required section moduli of web frames  $Z_{f,j,\text{rule}}(\mathbf{x})$ :

**Table 2**

Thickness of plates.

No.	Thickness $t$ , mm	No.	Thickness $t$ , mm
1	3.00	8	12.00
2	4.00	9	15.00
3	5.00	10	20.00
4	6.00	11	30.00
5	7.00	12	40.00
6	8.00	13	50.00
7	10.00	14	60.00

**Table 3**

Dimensions of bulb extrusions.

No.	Dimensions ( $h, b, s, s_1$ ) <sup>a</sup> , mm	Cross-sectional area, cm <sup>2</sup>
1	80 × 19 × 5 × 7.5	5.05
2	100 × 20.5 × 5 × 7.5	6.16
3	120 × 25 × 8 × 12	11.64
4	140 × 27 × 8 × 12	13.64
5	150 × 25 × 6 × 9	10.71
6	160 × 29 × 7 × 10.5	13.51
7	200 × 38 × 10 × 15	24.20

<sup>a</sup>  $h$  – cross-section height;  $b$  – flange width;  $s$  – web thickness;  $s_1$  – flange thickness.

$$Z_{f,j} - Z_{f,rule}(\mathbf{x}) \geq 0 \quad (5)$$

where  $Z_{f,j}$  – actual value of the section modulus of web frames in  $j$ -th region,– required shear area of stiffeners  $A_{t,s,j,rule}(\mathbf{x})$ :

$$A_{t,s,j} - A_{t,s,j,rule}(\mathbf{x}) \geq 0 \quad (6)$$

where  $A_{t,s,j}$  – actual value of shear area of stiffeners in  $j$ -th region,– required shear area of web frames  $A_{t,f,j,rule}(\mathbf{x})$ :**Table 4**

Dimensions of T-bulb extrusions.

No.	Sizes ( $h, b, s, s_1$ ) <sup>a</sup> , mm	Cross-sectional area, cm <sup>2</sup>
1	200 × 100 × 8 × 15	29.80
2	200 × 140 × 8 × 5	35.80
3	200 × 60 × 10 × 12	22.50
4	200 × 50 × 8 × 9.5	21.04
5	210 × 50 × 5 × 16	14.78
6	216 × 140 × 7.6 × 8	37.60
7	220 × 80 × 5 × 8	17.00
8	230 × 80 × 10 × 8	28.60
9	230 × 80 × 5.8 × 8	19.28
10	235 × 170 × 8 × 10	35.00
11	240 × 140 × 6 × 10	27.80
12	260 × 90 × 5 × 9.5	21.08
13	275 × 150 × 9 × 12	41.67
14	280 × 100 × 5 × 8	21.60
15	280 × 100 × 8 × 10	31.60
16	300 × 60 × 15 × 15	51.75
17	310 × 100 × 7 × 16	36.58
18	310 × 123 × 5 × 8	24.94
19	350 × 100 × 8 × 10	37.20
20	350 × 100 × 5 × 8	25.10
21	390 × 150 × 6 × 8	34.92
22	390 × 150 × 6 × 12	40.68
23	400 × 140 × 5 × 8	30.80
24	410 × 100 × 6 × 8	32.12
25	420 × 15 × 5 × 10	35.10
26	420 × 15 × 8 × 10	47.80
27	450 × 100 × 9 × 10	49.60
28	450 × 150 × 10 × 12	61.80

<sup>a</sup>  $h$  – cross-section height,  $b$  – flange width,  $s$  – web thickness,  $s_1$  – flange thickness.

**Table 5**

Simplified specification of bit representation of design variables as a chromosome substring.

No. $i$	Symbol $x_i$	Item	Substring length (no. of bits)	Value	
				Lower limit $x_{i,\min}$	Upper limit $x_{i,\max}$
1	$x_1$	Serial No. of mezzanine deck plate	4	1	10
2	$x_2$	Serial No. of mezzanine deck bulb	3	1	7
3	$x_3$	Serial No. of mezzanine deck T-bulb	4	42	52
4	$x_4$	Number of web frames	3	10	16
5	$x_5$	Number of mezzanine deck stiffeners	4	25	40
6	$x_6$	Serial No. of superstructure I plate	4	1	10
7	$x_7$	Serial No. of superstructure I bulb	3	1	7
8	$x_8$	Serial No. of superstructure I T-bulb	4	42	52
9	$x_9$	Number of superstructure I stiffeners	3	4	11
10	$x_{10}$	Serial No. of inner side plate	4	1	10
11	$x_{11}$	Serial No. of inner side bulb	3	1	7
12	$x_{12}$	Serial No. of inner side T-bulb	4	42	52
13	$x_{13}$	Number of inner side stiffeners	3	18	25
14	$x_{14}$	Serial No. of bottom plate	4	1	12
15	$x_{15}$	Serial No. of bottom bulb	3	1	7
16	$x_{16}$	Serial No. of bottom T-bulb	4	42	52
17	$x_{17}$	Number of bottom stiffeners	4	15	25
18	$x_{18}$	Serial No. of outer side plate	4	1	12
19	$x_{19}$	Serial No. of outer side bulb	3	1	7
20	$x_{20}$	Serial No. of outer side T-bulb	4	42	52
21	$x_{21}$	Number of outer side stiffeners	4	18	33
22	$x_{22}$	Serial No. of wet deck plate	4	1	12
23	$x_{23}$	Serial No. of wet deck bulb	3	1	7
24	$x_{24}$	Serial No. of wet deck T-bulb	4	42	52
25	$x_{25}$	Number of wet deck stiffeners	4	25	40
26	$x_{26}$	Serial No. of main deck plate	4	2	12
27	$x_{27}$	Serial No. of main deck bulb	3	1	7
28	$x_{28}$	Serial No. of main deck T-bulb	4	42	52
29	$x_{29}$	Number of main deck stiffeners	4	25	40
30	$x_{30}$	Serial No. of superstructure II plate	4	1	10
31	$x_{31}$	Serial No. of superstructure II bulb	3	1	7
32	$x_{32}$	Serial No. of superstructure II T-bulb	4	42	52
33	$x_{33}$	Number of superstructure II stiffeners	3	4	11
34	$x_{34}$	Serial No. of upper deck plate	4	1	10
35	$x_{35}$	Serial No. of upper deck bulb	3	1	7
36	$x_{36}$	Serial No. of upper deck T-bulb	4	42	52
37	$x_{37}$	Number of upper deck stiffeners	4	25	40
		Multivariable string length (chromosome length)	135		

$$A_{t,f,j} - A_{t,f,j,\text{rule}}(\mathbf{x}) \geq 0 \quad (7)$$

where  $A_{t,f,j}$  – actual value of the shear area of web frames in  $j$ -th region.

Side constraints  $h_i(\mathbf{x})$ , mathematically defined as equilibrium constraints, for design variables are given in Table 5. They correspond to the limitations of the range of the profile set. Some of them are pointed according to the author' experience in improving the calculation convergence.

The additional geometrical constraints were introduced due to "good practice" rules:

- assumed relation between the plate thickness and web frame thickness:

$$t_j - t_{f,w,j} \geq 0 \quad (8)$$

where  $t_j$  – actual value of the plate thickness in  $j$ -th region,  $t_{f,w,j}$  – actual value of web frame thickness in  $j$ -th region,

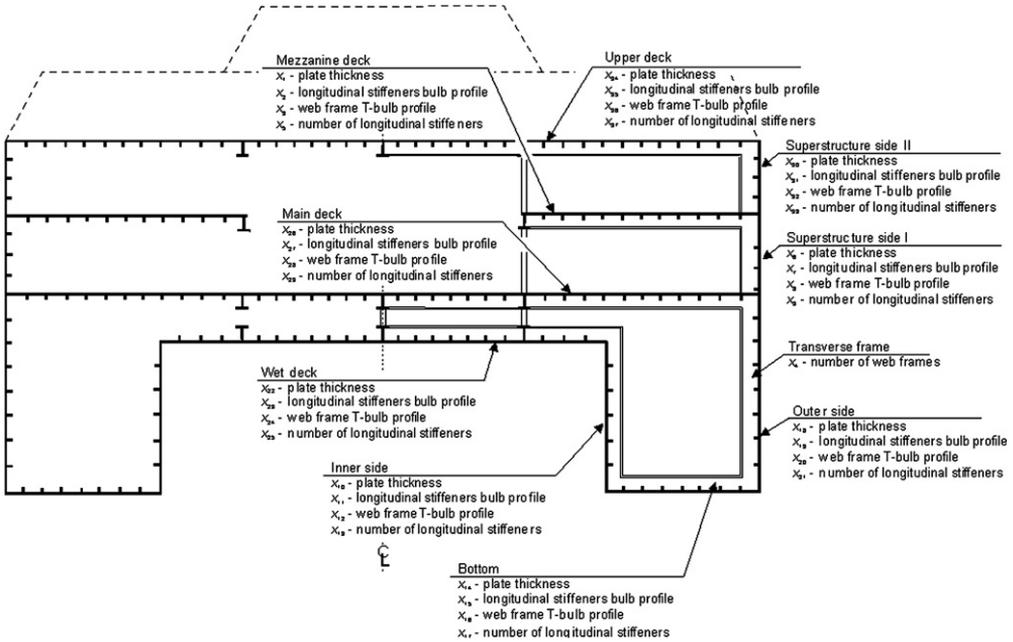


Fig. 5. Assumed model of craft – specification of design variables.

– assumed relation between the plate thickness and stiffener web thickness:

$$t_j - t_{s,wj} \geq 0 \tag{9}$$

where  $t_j$  – actual value of the plate thickness in  $j$ -th region,  $t_{s,wj}$  – actual value of stiffener web thickness in  $j$ -th region,

–assumed minimal distance between the edges of frame flanges:

$$l(x_4 + 1) - b_{fj} \geq 0.3 m \tag{10}$$

where  $b_{fj}$  – actual value of frame flange breadth in  $j$ -th region.

These relationships supplement the set of inequality constraints  $g_j(\mathbf{x})$ .

Finally, taking into consideration all specified assumptions, the optimization model can be written as follows:

- find vector of design variables  $(\mathbf{x}) = \text{col}\{x_1, \dots, x_i, \dots, x_n\}$ ,  $x_i, i = 1, \dots, 37$  as shown in Table 5,
- minimize objective function  $f(\mathbf{x})$  given by Eq. (2),
- subject to behaviour constraints given by Eqs. (3)–(7), side constraints given in Table 5 and geometrical constraints given by Eqs. (8)–(10) build a set of equality  $h_k(\mathbf{x})$  and inequality  $g_j(\mathbf{x})$  constraints.

## 6. Description of the genetic model

### 6.1. General

The topology and sizing optimization problem described in Sections 4 and 5 contains a large number of discrete design variables and also a large number of constraints. In such a case GA seems to be especially useful. Solution of the optimization problem by GA calls for formulation of an appropriate optimization model. The model described in Sections 4 and 5 was reformulated into an optimization

model according to requirements of the GA and that model was further used to develop suitable procedures and define search parameters to be used in the computer code.

The genetic type model should cover:

- definition of chromosome structure,
- definition of fitness function,
- definition of genetic operators suitable for the defined chromosome structures and optimization task,
- list of the searching control parameters.

## 6.2. Chromosome structure

The space of possible solutions is a space of structural variants of the assumed model. The hull structural model is identified by the vector  $\mathbf{x}$  of 37 design variables,  $x_i$ . Each variable is represented by a string of bits used as chromosome substring in GA. The simple binary code was applied. In Table 5 a simplified specification for bit representation of all design variables is shown. Such coding implies that each variant of solution is represented by a bit string named chromosome. Length of chromosome which represents structural variant is equal to the sum of all substrings. Number of possible solutions equals the product of values of all variables. In the present work the chromosome length is equal to 135 bits making the number of possible solutions approximately equal to  $10^{38}$ .

## 6.3. Fitness function

A fitness function is used to determine how the ship structure topology and sizing is suitable for a given condition in the optimum design with a GA. The design problem defined in previous parts of this paper is to find the minimum weight of a hull structure without violating the constraints. In order to transform the constrained problem into unconstrained one and due to the fact that GA does not depend on continuity and existence of the derivatives, so called “penalty method” has been used. The contribution of the penalty terms is proportional to violation of the constraint. In the method the augmented objective function of unconstrained minimization problem is expressed as:

$$\Phi(\mathbf{x}) = f(\mathbf{x}) - \sum_{k=1}^{n_c} w_k P_k \quad (11)$$

where  $\Phi(\mathbf{x})$  – augmented objective function of unconstrained minimization;  $f(\mathbf{x})$  – objective function given by Eq. (2);  $P_k$  – penalty term to violation of the  $k$ -th constraint;  $w_k$  – weight coefficients for penalty terms;  $n_c$  – number of constraints. Weight coefficients  $w_k$  are adjusted by trial.

Additionally a simple transformation of minimization problem (in which the objective function is formulated for the minimization) into the maximization is necessary for the GA procedures (searching of the best individuals). It can be done multiplying the objective function by  $(-1)$ . In that way, the minimization of the augmented objective function was transformed into a maximization search using:

$$F_j = \Phi_{\max} - \Phi_j(\mathbf{x}) \quad (12)$$

where  $F_j$  – fitness function for  $j$ -th solution;  $\Phi_j(\mathbf{x})$  – augmented objective function for  $j$ -th solution;  $\Phi_{\max}$  – maximum value of the augmented function from all solutions in the simulation. The value of parameter  $\Phi_{\max}$  has to be arbitrary chosen by a user of the software to avoid negative fitness values. Its value should be greater than the expected largest value of  $\Phi_j(\mathbf{x})$  in the simulation. In the presented approach the value  $\Phi_{\max} = 100.00$  was assumed.

#### 6.4. Genetic operators

The basic genetic algorithm (*Simple Genetic Algorithm* – SGA) produces variants of the new population using the three main operators that constitute the GA search mechanism: selection, mutation and crossover. The algorithm in present work was extended by introduction of elitism and updating.

Many authors described the selection operators which are responsible on chromosome selection due to their fitness function value [1,7,11,21]. After the analysis of the selection operators a roulette concept was applied for proportional selection. The roulette wheel selection is a process in which individual chromosomes (strings) are chosen according to their fitness function values; it means that strings with higher fitness value have higher probability of reproducing new strings in the next generation. In this selection strategy the greater fitness function value makes the individuals more important in a process of population growth and causes transmission of their genes to the next generations.

The mutation operator which introduces a random change of the chromosome was also described [1,21]. Mutation is a random modification of the chromosome. It gives new information to the population and adds diversity to the mate pool (pool of parents selected for reproduction). Without the mutation, it is hard to reach to solution point that is located far from the current direction of search, while due to introduction of the random mutation operator the probability of reaching any point in the search space never equals zero. This operator also prevents against the premature convergence of GA to one of the local optima solutions, thus supporting exploration of the global search space.

The crossover operator combines the features of two parent chromosomes to create new solutions. The crossover allows to explore a local area in the solution space. Analysis of the features of the described operators [1,11,21] led to elaboration of own,  $n$ -point, random crossover operator. The crossover parameters in this case are: the lowest  $n_{x\_site\_min}$  and the greatest  $n_{x\_site\_max}$  number of the crossover points and the crossover probability  $p_c$ . The operator works automatically and independently for each pair being intersected (with probability  $p_c$ ), and it sets the number of crossover points  $n_{x\_site}$ . The number of points is a random variable inside the set range [ $n_{x\_site\_min}$ ,  $n_{x\_site\_max}$ ]. The test calculations proved high effectiveness and quicker convergence of the algorithm in comparison to algorithm realizing single-point crossover. Concurrently, it was found that the number of crossover points  $n_{x\_site\_max}$  greater than 7 did not improve convergence of the algorithm. Therefore, the lowest and greatest values of the crossover points were set as following:  $n_{x\_site\_min} = 1$ ,  $n_{x\_site\_max} = 7$  (Table 6).

The effectiveness of the algorithm was improved with application of an additional updating operator as well as introduction of elitist strategy.

Random character of selection, mutation and crossing operators can have an effect that these are not the best fitting variants of the parental population which will be selected for crossing. Even in the case they will be selected, the result will be that progeny may have less adaptation level. Thus the efficient genome can be lost. Elitist strategy mitigates the potential effects of loss of genetic material copying certain number of best adapted parental individuals to progeny generation. In the most cases

**Table 6**  
Genetic model and values of control parameters.

No.	Symbol	Description	Value
1	$n_g$	Number of generations	5000
2	$n_i$	Size of population	2000
3	$n_p$	Number of pretenders (in elitist strategy)	3
4	$p_m$	Mutation probability	0.066
5	$p_c$	Crossover probability	0.80
6	$c\_strategy$	Denotation of crossover strategy (0 for fixed, 1 for random number of crossover points)	1
7	$n_{x\_site\_min}$	The lowest number of crossover points	1
8	$n_{x\_site\_max}$	The greatest number of crossover points	7
9	$p_u$	Update probability	0.33
10	$elitism$	Logical variable to switch on ( $elitism = yes$ ) and off ( $elitism = no$ ) the pretender selection strategy	yes

the elitist strategy increases the rate of dominating population by well-adapted individuals, accelerating the convergence of the algorithm. The algorithm selects fixed number of parental individuals  $n_p$  having the greatest values of the fitness function and the same number of descendant individuals having the least values of the fitness. Selected descendants are substituted by selected parents. In this way the operator increases exploitation of searching space. The number of pretenders  $n_p$  is given in Table 6. Update operator with fixed probability of updating  $p_u$  introduces an individual, randomly selected from the parental population, to the progeny population, replacing a descendant less adapted individual. The value of probability of updating  $p_u$  is also given in Table 6. This operator enhances exploration of searching space at the cost of decreasing the search convergence. It also prevents the algorithm from converging to a local minimum. Both operators acts in opposite directions, and they should be well balanced: exploitation of attractive areas found in the searching spaces as well as exploration of searching space to find another attractive areas in the searching space depends on the user's experience.

### 6.5. Control parameters

Single program run with the defined genetic model is characterized by values of 10 control parameters (Table 6).

For selection of more control parameters it is not possible to formulate quantitative premises because of the lack of an appropriate mathematical model for analysis of GA convergency in relation to control parameters. The control parameters were set due to test calculation results to achieve a required algorithm convergence; their values are presented in Table 6.

### 6.6. Conclusion

Finally, taking into consideration all specified assumptions, the genetic model can be written as follows:

- chromosome structure specified in Table 5,
- fitness function given by Eq. (12),
- genetic operators described in Section 6.4,
- control parameters specified and described in Section 6.5 and specified in Table 6.

## 7. Optimization calculations

To verify the correctness of the optimization procedure several test cases have been carried out using the model described in Sections 4–6. Each experiment is characterized by the 10 parameters, given in Table 6, controlling the search process. The set of experiment parameters are as follows:  $(n_g, n_i, n_p, p_m, p_c, c\_strategy, n\_x\_site\_min, n\_x\_site\_max, p_u, elitism) = (5000, 2000, 3, 0.066, 0.8, 1, 1, 7, 0.033, \text{yes})$ . A total number of  $10^6$  individuals were tested in the whole simulation. Results of typical search trial are presented in Tables 7 and 8 and Fig. 6.

The lowest value of the objective function,  $f(\mathbf{x}) = 4817.35$  kN, was found in the 868th generation. The corresponding values of design variables are given in Table 7.

From a designer's point of view these are structural weight and relative structural weight which are interesting values. They are given in Table 8.

The achieved structural weight value per volume unit is equal to  $0.23$  kN/m<sup>3</sup> and  $54.18$  kN/m for weight value per length unit. The higher value of structural weight per surface unit was for main deck region and equal to  $0.77$  kN/m<sup>2</sup>. It can be explained by high local loads of cars acting on this region. A lower value of structural weight per surface unit was achieved for superstructure regions being equal to  $0.24$  kN/m<sup>2</sup>. For the other structural regions intermediate values were obtained.

All values of the hull structural weight for feasible individuals searched in the simulation are presented in Fig. 7. The solid line represents the front of optimal solutions. It is composed of minimal (optimal) values of the structural weight received in the following simulation. All variants situated

**Table 7**  
Optimal values of design variables.

No.	Symbol	Description	Optimal value
1	$x_1$	Serial No. of mezzanine deck plate	5
2	$x_2$	Serial No. of mezzanine deck bulb	1
3	$x_3$	Serial No. of mezzanine deck T-bulb	49
4	$x_4$	Number of web frames	12
5	$x_5$	Number of mezzanine deck stiffeners	30
6	$x_6$	Serial No. of superstructure I plate	2
7	$x_7$	Serial No. of superstructure I bulb	4
8	$x_8$	Serial No. of superstructure I T-bulb	47
9	$x_9$	Number of superstructure I stiffeners	4
10	$x_{10}$	Serial No. of inner side plate	8
11	$x_{11}$	Serial No. of inner side bulb	4
12	$x_{12}$	Serial No. of inner side T-bulb	44
13	$x_{13}$	Number of inner side stiffeners	23
14	$x_{14}$	Serial No. of bottom plate	8
15	$x_{15}$	Serial No. of bottom bulb	6
16	$x_{16}$	Serial No. of bottom T-bulb	50
17	$x_{17}$	Number of bottom stiffeners	18
18	$x_{18}$	Serial No. of outer side plate	5
19	$x_{19}$	Serial No. of outer side bulb	1
20	$x_{20}$	Serial No. of outer side T-bulb	50
21	$x_{21}$	Number of outer side stiffeners	31
22	$x_{22}$	Serial No. of wet deck plate	5
23	$x_{23}$	Serial No. of wet deck bulb	1
24	$x_{24}$	Serial No. of wet deck T-bulb	50
25	$x_{25}$	Number of wet deck stiffeners	29
26	$x_{26}$	Serial No. of main deck plate	10
27	$x_{27}$	Serial No. of main deck bulb	3
28	$x_{28}$	Serial No. of main deck T-bulb	48
29	$x_{29}$	Number of main deck stiffeners	33
30	$x_{30}$	Serial No. of superstructure II plate	2
31	$x_{31}$	Serial No. of superstructure II bulb	4
32	$x_{32}$	Serial No. of superstructure II T-bulb	47
33	$x_{33}$	Number of superstructure II stiffeners	4
34	$x_{34}$	Serial No. of upper deck plate	2
35	$x_{35}$	Serial No. of upper deck bulb	3
36	$x_{36}$	Serial No. of upper deck T-bulb	43
37	$x_{37}$	Number of upper deck stiffeners	31

above the front of optimal solutions line are feasible but structural weight of these variants is greater than those situated on the front line. It can be seen how difficult it is to find the global optimum in the space search. Most of admissible variants created and evaluated during the simulation are remote from the global optimum and were used for the exploration of search space. A significant part of the computational effort is thus used by the algorithm for the exploration of search space and only a small part for exploitation of local optima. Such a “computational extravagance” is typical for all optimization algorithms employing random processes.

The graphs of the maximum, average, minimum and variance values of fitness across 5000 generations for simulation are presented in Fig. 8. The saturation was nearly achieved in this simulation. The maximum normalized fitness value is nearly 0.645. The standard deviation value is approximately constant and equal to 0.075 for all generations what means that heredity of generations is approximately constant over simulation. Variation of macroscopic quantities forming subsequential populations created throughout the simulation indicates evolutionally correct computations and that, for assumed values of the control parameters, it was not necessary to continue the simulation beyond assumed value of 5000 generations.

Quick stabilization of the mean value of the adaptation function and standard deviation indicates that considering the value of the adaptation function populations are homogenous in almost all

**Table 8**  
Optimal values of structural weight.

No.	Region description	Value, kN	Structural weight per surface unit, kN/m <sup>2</sup>
1	Mezzanine deck	487.22	0.31
2	Superstructure I + II	236.17	0.24
3	Inner side	489.67	0.44
4	Bottom	553.36	0.63
5	Outer side	481.46	0.31
6	Wet deck	463.82	0.39
7	Main deck	1581.25	0.77
8	Upper deck	524.40	0.29
9	Total weight, kN	4817.35	
10	Structural weight per volume unit, kN/m <sup>3</sup>	0.23	
11	Structural weight per length unit, kN/m	54.18	

simulation period. Quick stabilization of the mean value of the adaptation function as well as the standard deviation indicate that almost all populations are homogenous.

Evolution of the number of feasible individuals throughout the simulation is shown in Fig. 9. The number of feasible individuals found in the simulation increases with respect to time. It can be seen that the number of feasible variants is linearly dependant on the number of populations. In the whole simulation 1462 feasible individuals were found what approximately equals 0.015% of all checked individuals.

Evolution of fitness function values and the minimum values of structural weight are shown in Fig. 10. A correspondence of the diagrams can be seen. The increase of the fitness function values in successive generations is accompanied by the decrease of structural weight values. In the significant period of the simulation the algorithm used to find variants with better value of the fitness function, even so these were not variants having better values of optimization criterion – structural weight. Beginning with certain generation, the results become better not due to the value of the objective

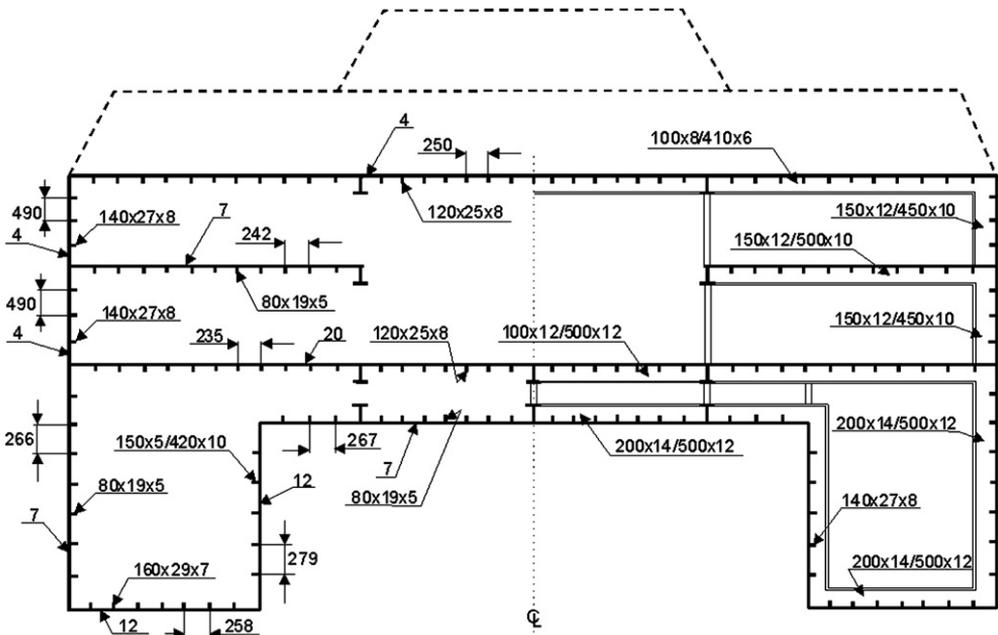
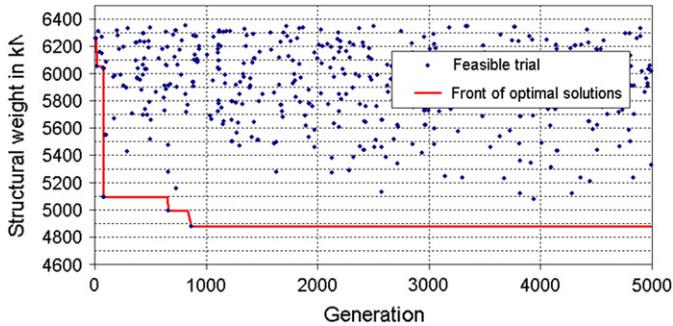


Fig. 6. Optimal dimensions and scantlings of vessel structure.



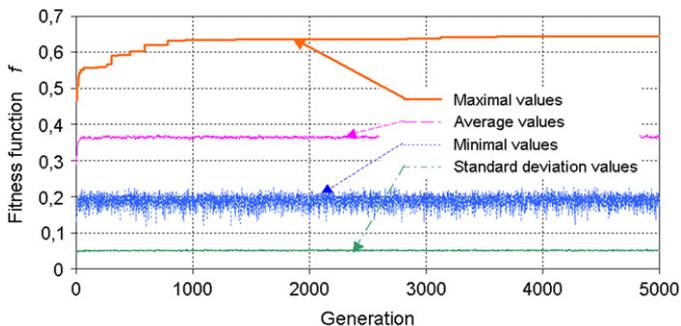
**Fig. 7.** Evolution of structural weight values over 5000 generations; solid line for absolutely minimal structural weight found during simulation (only feasible solutions are shown).

function but due to better fitting to constraints. Variation of the fitness function as well as the structural weight proves the correct course of the simulation considering the optimization of the structure with respect to its weight.

Both figures, Fig. 8 and 10, indicate quantitatively that the computer simulation realizing evolutionary searching for the solution of the topology-sizing in relation to weight of the ship structure optimization was successful and the final result can be taken as the solution of the formulated optimization problem. As it is known, the conclusion cannot be confirmed by precise mathematical methods.

Number of all possible variants in the genetic model, number of checked individuals over 5,000 generations and number of feasible individuals checked over simulation are shown in Fig. 11. Presented values show how much computational effort is used to find a small number of the feasible variants among which we expect the optimum variant be located. It seems that it is a cost we should accept if we want to keep the high ability of the algorithm to explore of the solution space. Retaining the values of another control parameters, the number of the feasible variants can be increased adjusting variation ranges of the design variables. The ranges can be either narrowed or shifted towards larger values of the design variables so that it is easier to obtain feasible variants. In each specific case the selection of the strategy is dependant on the user:

- whether to allow the wider searching solution space expecting solutions closer to optimum can be found at the expense of longer computational time,
- or to decrease the computational time accepting that the solutions will be more remote from the optimum.



**Fig. 8.** Evolution of maximum, average, minimum and standard deviation values of the fitness over 5000 generations; fitness function values are dimensionless and normalised to produce extreme value equal to 1.0.

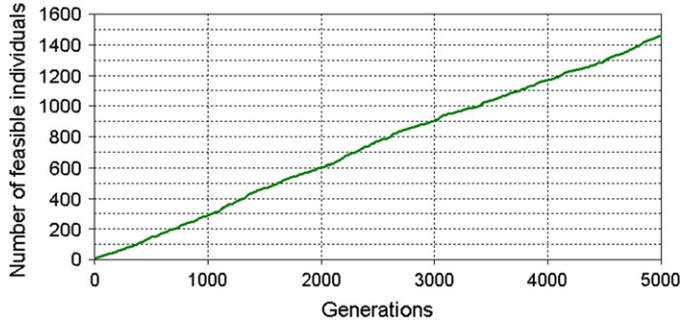


Fig. 9. Evolution of number of feasible individuals over 5000 generations; 1462 feasible individuals have been checked.

Methodology of scientific investigation requires that the quantitative results be verified. In this case the verification can be performed either (i) comparing to appropriate values of a real structure or (ii) comparing to the recognized results of comparable computations performed by other authors. Concerning (i) the author does not have corresponding data since the shipyards usually do not publish the data on structural weight. Concerning (ii) it should be remarked that similar optimization problems referring to ship structures are rarely undertaken by the other authors therefore the examples are unique or unpublished. Specifically, the author does not have a reference data on the structural model taken for the investigation. In this context the presented investigation does not answer the question whether the obtained results indicate a possibility to design the structure lighter than actual but the existence of a method which is applicable for solution of the unified topology-size optimization for a seagoing ship structure in more general sense.

The investigation carried out within the present paper confirmed the three unquestionable advantages of GA which make them attractive and useful for optimization of ship structures: (i) resistance to existence of many local extremes in the search space, (ii) lack of necessity of differentiation of the objective and limit functions and (iii) easiness of modeling and solution of the problems involving discrete variables. Of course they also have disadvantages, the most important being: (i) computational extravagance (large computational cost used for exploration of the search space) and (ii) lack of formal convergence criteria. Additional advantages which can decide perspective on the more common use of the algorithms are: (i) existence of developed and published algorithms of multi-criteria optimization as well as (ii) effective computations on networks of computers or multi-processor computers.

Significant computational costs required for ship structural optimization employing GA cause, at the present speed of commercial computer systems, strong doubts on possibility of application of direct methods of structural analysis for estimation of behaviour constraints. It seems that direct

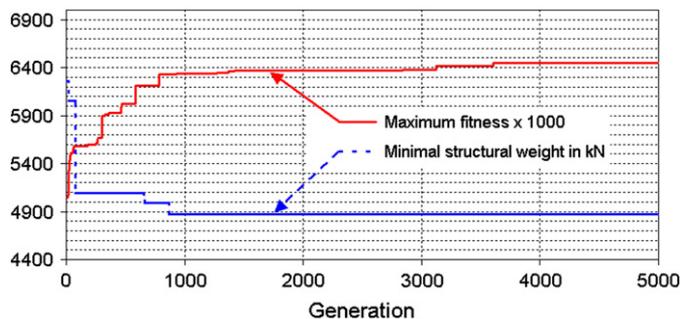
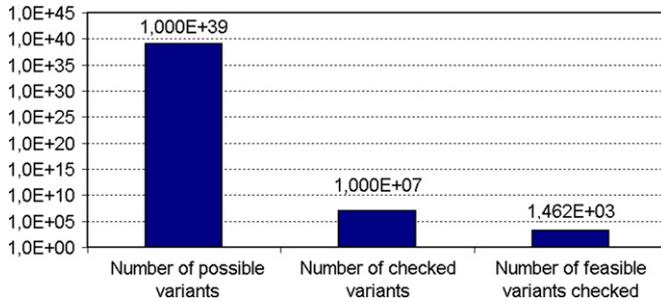


Fig. 10. Evolution of maximal fitness value and absolutely minimal structural weight over 5000 generations; absolutely minimal structural weight for simulation only for feasible solutions.



**Fig. 11.** Number of all possible variants, number of checked individuals over 5000 generations and number of feasible individuals checked over simulation.

application e.g. regularly used in practice the finite element method for the analysis of millions or even billions variants checked in the exploration of decision space seems impossible. Especially in the preliminary designing where many optimization investigations are to be performed in short time. In such a situation methods can be searched to limit the number of such calculations to e.g. preselected variants. A hybrid system can also be proposed where e.g. GA will allow to create, using fast rule equations or simplified methods of analysis, a set of variants localized in the vicinity of extremum and then searching the optimum solution using the selected analytical method using the direct methods of structural analysis.

## 8. Conclusions

The application of the genetic algorithm concept to solve the practical design problem of the optimization of hull structures of high speed craft was presented. The problem of weight minimization for a three-dimensional full midship block-section of the high speed catamaran hull was described.

In the study the design problem was limited to the minimization of the hull structural weight but it can be easily extended to include other criteria such as production cost what will be a subject of the further studies.

It was proven in the study that the GA allows to include in the optimization model a large number of design variables of the real ship structure. Introducing constraints related to strength, fabrication and standardization is not difficult and may cover a more representative set of criteria.

Simultaneous optimization of topology and scantlings is possible using the present approach. Enhancement of the sizing optimization (the standard task of the structural optimization) to allow for the topology optimization requires disproportional computational effort. It is an effect of both increase of the search space by introducing design variables referring to the structural topology as well as increase of number of generations and number of individuals to ensure satisfactory convergence of the optimization process.

Additionally the GA realization described in the paper is also under continuous development directed towards implementation of other genetic operators, genetic encoding, multi-objective optimization, etc. as well as including some other constraints.

Practical application of the GA to ship structural optimization calls for significant limitation of an optimization problem in the way of spatial delimitation of the structural region subject to optimization and/or limitation of variation of design variables.

As the final conclusion it can be said that the study confirmed that proposed realization of the GA presented a potentially powerful tool for optimizing the topology and sizing of ship hull structures.

The present paper is a successful attempt of unification of problems of topology and sizing optimization of ship structure and their solution using the GA. It was proven that the GA can be considered as a good method for solution of more general unified shape–material–topology–sizing optimization problems.

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