

# Ship Hull Structural Multiobjective Optimization by Evolutionary Algorithm

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An evolutionary algorithm for multiobjective optimization of the structural elements of the large spatial sections of ships is presented. The evolutionary algorithm where selection takes place based on the aggregated objective function combined with domination attributes as well as distance to the asymptotic solution is proposed and applied to solve the problem of optimizing structural elements with respect to their weight and surface area on a high-speed vehicle–passenger catamaran structure with several design variables such as plate thickness, scantlings of longitudinal stiffeners and transverse frames, and spacing between longitudinal and transversal structural members. Details of the computational models were at the level typical for conceptual design. Scantlings were analyzed using the selected rules of a classification society. The results of numerical experiments with the use of the developed algorithm are presented. They show that the proposed genetic algorithm can be a foundation of the effective multiobjective optimization tool for ship structure optimization. Further development of the tool should include more advanced methods for ship structural analysis.

**Keywords:** ship structure; multiobjective optimization; evolutionary algorithm; genetic algorithm; Pareto domination; set of nondominated solutions

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## 1. Introduction

GENERALLY SPEAKING, ship structural design consists of selection and spatial arrangement of material in the form of structural components (decks, bulkheads, hull sections) composed of secondary structural elements (stiffeners, plating, etc.). The ship hull should provide safe ship operation at the least costs.

Strategic decisions (regarding for example the selection of the structural material, frame spacing, layout of the cargo space, layout of the fuel, and ballast tanks) fundamental for meeting the design objectives (e.g., cargo deadweight, fuel consumption) are taken in the stage of the concept design because in this stage, the principal ship properties are decided and it is here where the risk of making fundamental errors in designing the vessel can occur. Correcting these errors in the later stages of the design process can be very costly or even

impossible. The investigation presented here refers to the ship concept design.

Considering dimensioning of structural elements, the basis for the structural design are either requirements of classification societies published as classification rules, containing simplified formulas for evaluating structural loads and dimensioning structural elements resisting these loads, or a more rational approach based directly and fully on the structural mechanics. Regardless of occasionally formulated criticism of the classification rules as a tool not meeting the designer's expectations in the case of designing innovative solutions, they are, however, a set of recommendations derived from the good practice as well as scientific investigations and their application in case of conventional ships is fully satisfactory. In the present article, it was assumed that dimensions of structural elements are determined only according to the requirements of a classification society enabling quick and automatic dimensioning of many structural variants by the optimization algorithm.

A primary task of the ship structural optimization is to find the optimal space localization as well as scantlings of structural

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elements for the objective function subject to constraints. Formally, selection of the structural material can also be treated as part of the optimization process; nevertheless, selection of the structural material is usually not an explicit optimization task but is rather done according to the experience and capability of a shipyard. Systematic optimization procedures for the selection of structural material are applied directly in rare cases. In this article, the problem of optimization of the structural material is also not addressed.

Moreover, the design of such a complex object as ship structure is a solution of the multiobjective optimization task including many optimization objectives often counteracting each other, e.g., small hydrodynamic resistance versus large cargo deadweight and high structural strength and reliability versus low structural weight. This requires a comprehensive search of the solution space without the capability to select one solution unequivocally selected as the best one, because it is in the single-objective optimization tasks. This is because multiobjective optimization does not yield an unequivocal determination of the single variant proposed for further development, but a set of compromise solutions (infinite in general), which is used as a basis for taking the final design decision consisting of selection of a solution (or solutions) to be further developed. A task of the multiobjective optimization is thus the appropriate identification of the set of “best possible compromises” or the single “best possible solution” as a result of the multiobjective ship structure design process.

As a result of high complexity, despite rising research and computational resources, the multiobjective optimization of the ship structures is still held back by a number of obstacles hindering its application in practice, and the attempts to solve the problem can be judged as marginal. Most authors assume that an outcome of the multiobjective optimization task is a set of the Pareto-optimal solutions, whereas it is impossible to point the best among them objectively. Classic, mostly based on aggregation of objective functions multiobjective optimization algorithms allows for finding in the best case a single solution in the single algorithm run, which makes them unsuitable for the multiobjective optimization tasks involving the determination of the Pareto-optimal solution set. At the same time, the evolution-based algorithms for example allow for determination of this set in the single algorithm run thanks to the fact that they process not single solutions, but usually the large set of potential solutions, which in the consecutive steps gradually evolve to a Pareto-optimal set.

The evolutionary algorithm was proposed in this article to the optimization of the seagoing ship structure using in the process of the selection of the combined fitness function including in one mathematical expression: 1) optimization objectives; 2) penalty function for constraints violation; 3) domination attributes (dominance rank as well as dominance count); and 4) distance to the asymptotic solution. A practical example of the application of the developed algorithm is presented, featuring the multiobjective optimization of the structure of fast passenger-vehicle ferry concept named Auto Express 82m. A task of the two-objective optimization of the ship structure minimizing its weight and surface area for cleaning and painting has been formulated. A number of optimization objectives has been limited to two for the sake of simplicity of graphical presentation of results and their analysis. The precision of the

developed computational model has been limited to the level typical for the preliminary phases of the design process for structures of a similar type. The formulated constraints account for structural strength values estimated using procedures laid down in classification rules. A computation application has been built for the solving of this formulated task, being based on a specialized genetic algorithm code. Appropriate models of the ship structures have been built and computational investigations carried out. The obtained results have led to the conclusion that the presented algorithm can be considered as a method allowing for the solution of the multiobjective optimization problems formulated for the ship structures in the concept design stage. The further development of the proposed method should include more advanced methods for structural analysis.

## 2. Basic concepts of multiobjective optimization

From a mathematical point of view, the multiobjective optimization can be defined in the general way as a procedure consisting of selecting an element of the set on the basis of relations establishing some order in this set. In reference to the ship structural design, the elements of this set are in general the representations of particular problem solutions such as ship structural variants. This set, referred to as “set of possible solutions,” is a subset of solution space,  $V_x$ . As we know, the set of such solutions is limited by the introduction of various constraints and such a constrained set is then called “set of feasible solutions  $\Phi$ .” For obvious reasons, a set of feasible solutions  $\Phi$  is also subset of solution space  $V_x$ , and each element of this space is a vector of design variables  $\mathbf{x} \in V_x$ . Solution space,  $V_x$ , may be a functional space or numerical space, if all its coordinates are numbers:

$$\mathbf{x} = [x_1 \ x_2 \ \dots \ x_i \ \dots \ x_n]^T \in V_x \quad (1)$$

The case where the solution space is a  $n$ -dimensional numerical space,  $\mathfrak{R}^n$  is most often encountered in practical applications. In the further part of the article, the solution space is an  $\mathfrak{R}^n$  space.

Solution of the multi-objective optimization problem can be formulated in the following way: find a combination of design variable values  $\mathbf{x} = [x_1 \ x_2 \ \dots \ x_i \ \dots \ x_n]^T$ , which optimizes at the same time all components of a given objective function vector  $\mathbf{f}(\mathbf{x}) = [f_1(\mathbf{x}) \ f_2(\mathbf{x}) \ \dots \ f_s(\mathbf{x}) \ \dots \ f_n(\mathbf{x})]^T$ . With a possibility to impose constraints on the variability ranges of design variables. It is also assumed that all the functions occurring within the problem are real ones, and the number of constraints is finite. Taking into account the demand or computational resources and their cost, another requirement may be formulated, that the selection made could be implemented at the lowest possible cost. Exact definition of the meaning of term “optimize” has crucial significance in case of multi-objective optimization problem. In the further part of the paper this concept is going to be discussed in more detail. The general mathematical formulation of a multi-objective optimization problem can be presented as follows:

$$\begin{aligned} &\text{for design variables } \mathbf{x} = [x_1 \ x_2 \ \dots \ x_i \ \dots \ x_n]^T: \\ &x_{i,min} \leq x_i \leq x_{i,max}, \quad i = 1, 2, \dots, n, \end{aligned} \quad (2a)$$

$$\text{minimize}^a \mathbf{f}(\mathbf{x}) = [f_1(\mathbf{x}) f_2(\mathbf{x}) \dots f_s(\mathbf{x}) \dots f_S(\mathbf{x})]^T$$

$$s = 1, 2, \dots, S, \quad (2b)$$

$$\text{subject } h_k(\mathbf{x}) = 0 \quad k = 1, 2, \dots, K, \quad (2c)$$

$$g_j(\mathbf{x}) \geq 0 \quad j = 1, 2, \dots, J \quad (2d)$$

where  $\mathbf{f}(\mathbf{x})$  is an objective function vector having  $S$  optimization objectives,  $\mathbf{x}$  is an independent variable vector having  $n$  design variables,  $h_k(\mathbf{x})$  and  $g_j(\mathbf{x})$  are constraint functions, and  $x_{i,\min}$  and  $x_{i,\max}$  upper and lower limit of variables, respectively. Equations  $h_k(\mathbf{x}) = 0$  are referred to as equality constraints, whereas inequalities  $g_j(\mathbf{x}) \geq 0 =$  inequality constraints.

A solution of multiobjective decision-making problem, originally formulated by Francis Ysidro Edgeworth in 1881 (Edgeworth 1881) and generalized then by Vilfredo Pareto (Pareto 1896) is the commonly accepted measure of quality in the multiobjective selection problems. It is now referred to as Edgeworth-Pareto optimum or Pareto optimum. According to the definition, a design of the ship structure may be referred to as Pareto-optimal under the condition that there are no other variants of the structure that are better with regard to at least one objective while at the same time being equally good with regard to all the other optimization objectives. This means that the Pareto-optimal structural variant cannot be improved without simultaneous worsening of at least one objective. The variant of the ship structure is not Pareto-optimal if there is any other variant improving at least one objective while at the same time not worsening the values obtained for the remaining ones. Such variants are also referred to as dominated ones or inferior ones.

Using the concept of domination formulated by Pareto, we can say that a multiobjective optimal solution is each solution that has no other feasible solutions dominating it. We say that the solution  $\mathbf{x}_1$  dominates (is better than) the solution  $\mathbf{x}_2$  if the following two conditions are satisfied:

$$f_s(\mathbf{x}_1) \leq f_s(\mathbf{x}_2), \text{ for all } s = 1, 2, \dots, S, \text{ and} \quad (3a)$$

$$f_{s'}(\mathbf{x}_1) < f_{s'}(\mathbf{x}_2), \text{ for at least one } s' = 1, 2, \dots, S \quad (3b)$$

There is then no such a solution in the set of feasible ones for which the value of all objectives would be “better” than their respective values for any multiobjective optimal one. In other words, the multiobjective optimal solution is a feasible solution for which no better solution can be found in the set of feasible solutions. The term “better” should be understood here in the sense of Pareto domination.

The concept of Pareto domination allows for introduction of the two-value measure of quality for solutions of the multiobjective optimization problem. It allows for dividing the set of feasible solutions into two subsets: 1) subset of dominated solutions and 2) subset of nondominated solutions, which may be considered to

be the solution of a multiobjective optimization problem. The two-value of this measure does not allow for a further evaluation of a feasible dominated solutions set<sup>b</sup> and, particularly, does not allow for relative estimation of distances between dominated solutions and the set consisting of nondominated solutions (set of Pareto-optimal solutions) by any other feasible solution. Despite this, the relation of Pareto domination is the one most often used for the definition of multiobjective optimal solution. In further part of this work, when talking about domination relation, we shall then understand it to be the relation of Pareto domination and the earlier used phrase “optimize vector objective function  $\mathbf{f}$ ” shall be understood as a command: find the Pareto-nondominated solutions within the feasible solutions set.

A basic feature of multiobjective optimal solutions is the fact that there are many (or even an infinite number) of them existing in practical problems. In the case of the continuous feasible set, being a subset of  $\mathfrak{R}^n$ , the set of feasible objectives shall also be continuous and a subset of  $\mathfrak{R}^2$  (as a result of two quality criteria). As a result of the analysis of feasible objectives set, we can obtain not several multiobjective optimal points, but the whole curve of multiobjective optimal objectives, presented in Fig. 1.

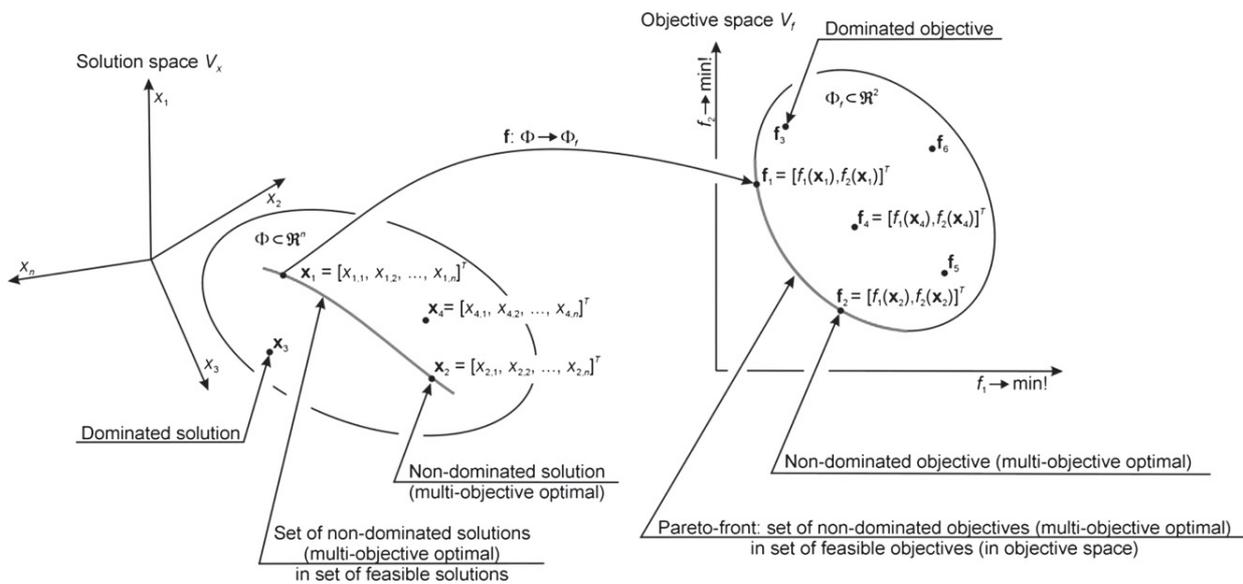
### 3. Solving of a multiobjective optimization problem: how to find a set of nondominated solutions

Monographs and books on the general problems of multi-objective optimization are, e.g., Cohon (1978), Stadler (1988), Eschenauer et al. (1990), Statnikov and Matosov (1995), Miettinen (1998), and Sen and Yang (1998). Rare works focusing on multiobjective optimization of ship structure include Moe and Lund (1968), Shi (1992), Das et al. (1993), Ray and Sha (1994), Sen and Yang (1995), Jianguo and Zuoshui (1996), Parsons and Scott (2004), Shin et al. (2006), Jang et al. (2009), Jelovica and Klanac (2009), and Caprace et al. (2010).

Many methods of classification of the multiobjective optimization problems can be found in the literature as well as methods of solution of such problems dependent on the classification criterion. Assuming that the multiobjective optimization problems can be primarily classified with respect to the sources of inspiration of the methods, they can be divided into the two following categories: 1) classic methods and 2) methods inspired by natural systems, particularly evolutionary methods. The classic methods include two basic methods used for the solving multiobjective optimization tasks: (1.1) optimization problems are solved with regard to all optimization objectives taken individually one by one while the remaining objectives are included in the set of constraints; (1.2) a substitute scalar objective function is formed of the adopted ones as a linear combination of the original component objectives multiplied by the appropriately selected weight coefficients and then the optimization problem is solved with regard to such a newly formed aggregate objective. For case (1.2), a series of calculations is usually carried out for various values of the weight coefficients, and the best among the found solutions is taken as the solution of the problem. The

<sup>a</sup>General definition of a multiobjective optimization problem was formulated assuming that the minima should be found for all of the optimization objectives. In the case when for a specific problem maxima single or more objectives should be found, multiplying them by  $-1$  is an easy way of transforming the problem of maximization of optimization objectives into minimization.

<sup>b</sup>That means the most detailed analysis of set of feasible solutions in relation to domination. Relationship of domination does not “see” many details of the objective space. The only details it can “see” whether the solution is dominated or nondominated.



**Fig. 1** Graphical illustration of line of nondominated objectives in the objective space and set of nondominated solutions in solution space;  $\mathbf{f}$  = vector objective function,  $\Phi$  = set of feasible solutions,  $\Phi_f$  = set of feasible objectives;  $f_1 \rightarrow \min!$ ,  $f_2 \rightarrow \min!$

methods based on a aggregation of the vector objective function have been used in wide-ranging applications also in the methods of evolution-based multiobjective optimization, because they allow for the use of well-developed single-objective optimization algorithms. Fundamental disadvantages of methods from this group are: 1) seeking only a single point on the non-dominated solutions front and resulting in a necessity to make numerous calculation runs for the single optimization task; 2) sensitivity of some solutions to the shape of nondominated solutions front; and 3) the fact that expert knowledge is required at the beginning to specify the weight coefficients used for component optimization objectives.

Classic methods used for the solving of multiobjective optimization problems based primarily on the aggregation of vector objective functions are easy to implement but ineffective in many cases. However, evolutionary multiobjective optimization algorithms developed in recent years have been proven to be highly effective (Deb 2001; Osyczka 2002; Sarker & Coello Coello 2002; Abraham et al. 2005; Coello Coello et al. 2007).

Not numerous but highly promising results in the field of genetic algorithms use for multiobjective optimization tasks have been obtained lately including also the results in the field of ship structures (Okada & Neki 1992; Hutchinson et al. 1998; Kitamura et al. 2000; Klanac et al. 2009; Sekulski 2010). Jang and Shin (1997) have applied the evolutionary strategy method for the multiobjective optimization of ship structures.

Special evolutionary multiobjective optimization methods can also be applied as far as genetic algorithms are concerned: VEGA—Schaffer (1985), HPGA—Hajela and Lin (1992), FFGA—Fonseca and Fleming (1993), NPGA—Horn et al. (1994), NSGA—Srinivas and Deb (1995), MOGA—Murata and Ishibuchi (1995), MOBES—Binh and Korn (1997), SPEA—Zitzler and Thiele (1998), MOMGA—Van Veldhuizen (1999), PAES—Knowles and Corne (1999), NSGA-II—Deb et al. (2000), and SPEA2—Zitzler et al. (2001). Fundamental advantages of these

methods are: 1) effective search of solution space and 2) capability to illustrate the nondominated solutions front in a single simulation run. Excellent presentation of evolutionary methods of multiobjective optimization can be found in recently published monographs (Deb 2001; Jaszkiewicz 2001; Osyczka 2002; Coello Coello et al. 2007). The principal elements of these algorithms are: 1) selection strategies based on the Pareto-domination relation; (2) niching strategies to preserve diversity in the consecutive populations; and (3) elitist strategy to ensure survival of non-dominated solutions in the time of evolution.

Classic methods of multiobjective optimization used the concept of scalarization of the optimization objectives in the first place. The first evolutionary algorithms adopted the concept in the natural way, yet when in the result of further investigation, efficient multiobjective evolutionary algorithms using various methods for classifying solutions with regard to the Pareto domination relation (where the scalar objective function is not evaluated) were found more efficient. Nevertheless, the researchers have reported for several years that if the number of the optimization objectives is greater than three, the methods based on the domination relation turn to be ineffective because together with the increase of the number of optimization objectives the number of nondominated variants decreases reducing the effectiveness of the selection operator (Hughes 2003, 2005; Purshouse & Fleming 2003; Jaszkiewicz 2004). The aggregation methods have been found promising again with the hope to: 1) develop more simple and intuitive algorithms than algorithms based on the domination relation, obtaining expertise on the multiobjective ship structural optimization; and 2) develop effective algorithms for problems with a large number of the optimization objectives. Of course, it needs to be considered what number of objectives is practically justified. Because at the end of the multiobjective decision process (even computer-aided), a responsible person or a group of responsible people with regard to capability of processing data by human beings and capability to work out decisions, it seems that the

maximum number of the optimization objectives in practical problems should be between five and seven.

As a result of the following practical problems: 1) lack of information about the actual localization of nondominated solutions and 2) necessity to deploy significant computational resources to solve the multiobjective optimization problem, the main effort in the practical evolution-based multiobjective optimization is directed at determining the acceptable approximation of the Pareto set instead of accurate composition of this set. With regard to this, it can be assumed that in practice, the result of a multiobjective optimization process is a set of nondominated solutions called shortly the approximation of Pareto set and not the exact Pareto-optimal solutions set. Practical formulation of the multiobjective optimization problem and of attained results should follow this guideline.

We note that opposite to the single-objective optimization problems where the objective and fitness functions are often identical, in the evolution-based algorithms for the multiobjective optimization, these functions should be distinguished. The optimization objectives should be used for defining the fitness function, which is a key factor for the selection process. From this point of view these methods can be in general divided with respect to the type of the fitness function used for calculations as follows: 1) selec-

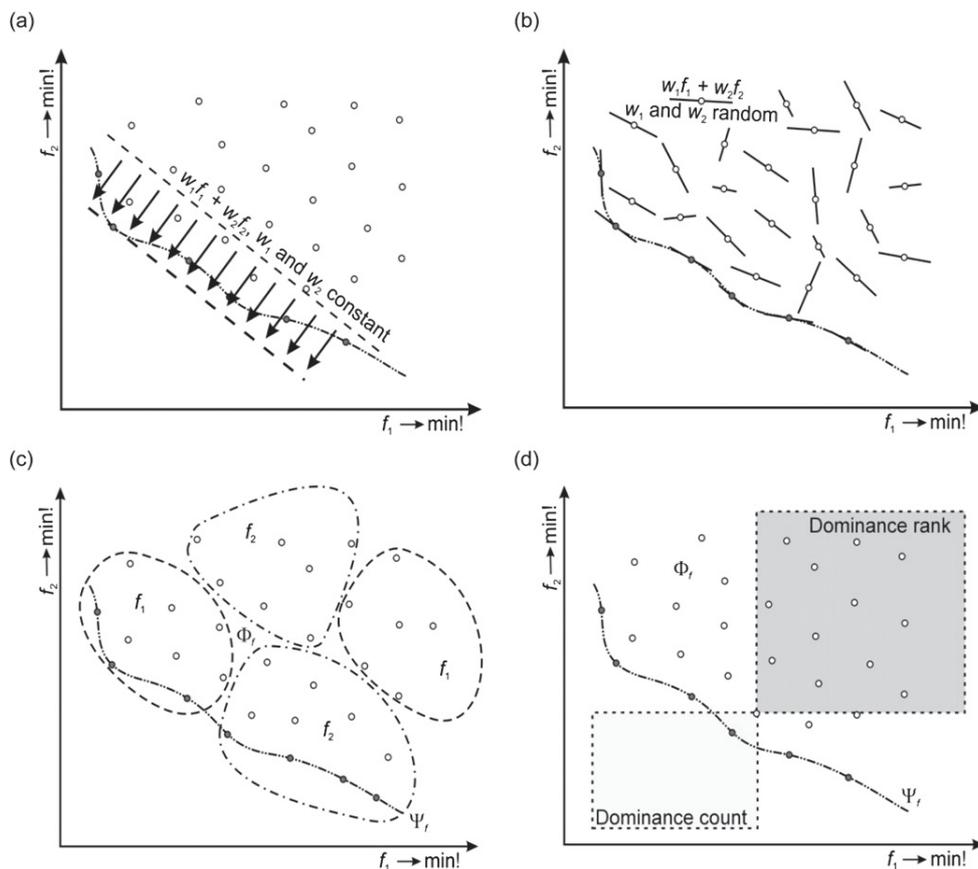
tion with respect to the scalar objective function with fixed weights of optimization objectives; 2) selection with respect to the scalar objective function with random weights of optimization objectives; and 3) division of the variant set into subsets and selection in each of them with respect to single objective (Fig. 2).

The first proposal, illustrated in Fig. 2a, stemming from classic methods used for the determination of compromise surface, consists of summing the objectives up and formulating a single, parameterized objective function. Parameters of this substitute objective function are fixed during the optimization run, which allows for finding the one nondominated solution. The multiobjective optimization problem is reduced to the single-objective problem. The simplest concept is the introduction of substitute objective function  $F$  as a linear combination of  $S$  partial optimization objectives  $f_s$ :

$$F(\mathbf{x}) = \sum_{s=1}^S w_s f_s(\mathbf{x}) \quad (4)$$

where  $w_s$  are coefficients determining the weights given to particular objectives.

The next proposals, illustrated on Fig. 2b and Fig. 2c, are based on the weighted sum of optimization objectives, where



**Fig. 2** Graphical illustration of selected strategies for taking into account the particular optimization objectives used in the multiobjective optimization algorithms: (a) selection with respect to the substitute scalar objective function with fixed weights of optimization objectives; (b) selection with respect to the substitute scalar objective function with random weights of optimization objectives; (c) division of the variant set into subsets and selection in each of them with respect to single objective; (d) selection with respect to attributes of Pareto domination;  $f_1 \rightarrow \min!$ ,  $f_2 \rightarrow \min!$

weight coefficients represent the values changing in the process of evolution (Hajela & Lin 1992; Ishibuchi & Murata 1996). Weight coefficients of the substitute objective function are change in the specific way during the optimization run, which allows for finding the nondominated solutions set instead of a single compromise solution.

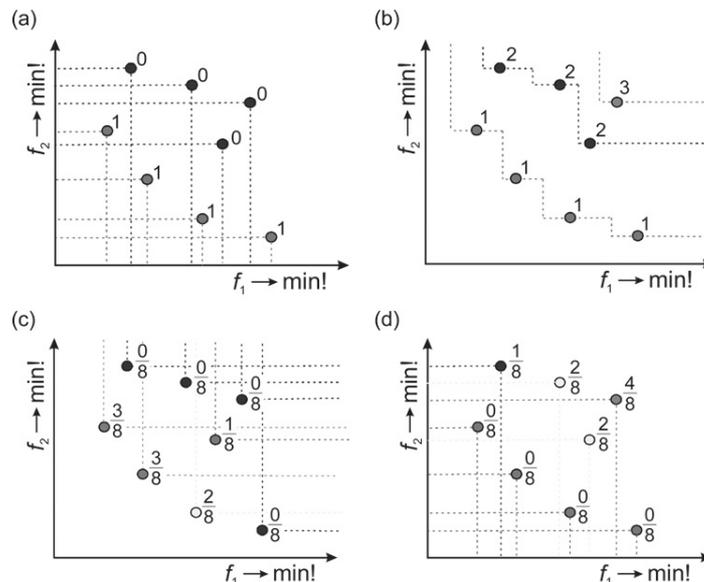
Methods based on selection with respect to the scalar objective function with random weights of optimization objectives (Fig. 2b) use numerical procedures for setting random values of weight coefficients  $w_s$ . The simplest and most frequently applied implementation of the method is setting random values of uniform distribution in the range [0, 1].

Methods based on selection according to a single objective as illustrated in Fig. 2c include mechanisms of switching between the objectives during the selection phase. In each case where the algorithm starts the execution of reproduction, some objective (potentially other) decides which member of the population is going to be copied to the set of variants earmarked for crossbreeding. For example, Schaffer (1985) proposed an algorithm, where the population is divided in advance to identical parts and then a different, single objective is used on the members of each of groups one by one and Kursawe (1991) proposed a different method consisting of the random selection of a single optimization objective to be used in the next step of selection process with probabilities to be set by the user or randomly adjusted during the evolution.

Assigned to feasible variants two-argument domination attribute is a rather general information and does not refer to inner structure of feasible objectives set (Fig. 3a). A detailed study of this set structure enables the use of additional knowledge and elaboration of very refined and useful tools supporting desirable convergence of a computing algorithm. Different detailed solutions using this concept can be found in the literature, for example: Fonseca and Fleming (1993), Zitzler and Thiele (1999), Deb

(2001), Osyczka (2002), Abraham et al. (2005), and Coello Coello et al. (2007).

The concept of calculating the fitness of variants while accounting for the Pareto-type domination (Fig. 2d) has already been proposed by Goldberg (1989) and since then taking the advantage of partial order being present in the population (Fig. 3b) has been facilitated by many various methods developed specifically for this purpose. Based on the concept of dominance depth proposed by Goldberg (1989), the feasible variants appearing in the consecutive generations are divided into consecutive fronts of nondominated variants located deeper and deeper in the feasible objectives set, or in other words, further and further from the Pareto front. Other proposals are based on the concept of dominance rank (Fig. 3c), which is based on use of the number of variants dominated by a selected variant for the calculation of fitness function value (Fonseca & Fleming 1993). Also a dominance count (which is a number of variants dominating a selected variant; Fig. 3d) may be taken into account. For example SPEA (Zitzler & Thiele 1999) and SPEA2 (Zitzler et al. 2001) strategies for the calculations of fitness function are based on both approaches, i.e., the rank and the dominance count. Disregarding the specific strategy applied, in case of these methods, the value of fitness depends on the characteristics of the variants remaining in the population in contrast with strategies based on aggregation of objectives or the selection of variants on the basis of less or more arbitrary selected single objective, where the fitness values are independent from the characteristics of other variants in the population. The multiobjective evolution-based optimization algorithms outlined previously have been tested by other authors on simple problems of multiobjective optimization, e.g., Zitzler (1999), Zitzler et al. (1999), and Zitzler and Thiele (1999). Because no systematic research into the suitability of these algorithms for the solving of optimization problems involved in the



**Fig. 3** Graphical illustration of the groups of dominance attributes: (a) classifying the feasible variants basing on Pareto-domination relation; (b) classifying the feasible variants basing on domination depth; (c) dominance rank (proposed strategy for the determination of a number of variants dominated by a given variant) in the objective space; and (d) dominance count (proposed strategy for the determination of a number of variants dominating a given variant);  $f_1 \rightarrow \min!$ ,  $f_2 \rightarrow \min!$

design of seagoing ship structures has been carried out so far, the application of a particular method should be preceded by systematic investigations into its effectiveness in the problems involved in the design of such structures.

#### 4. Looking inside the feasible objectives set: dominance rank and dominance count

Assignment of the two-argument dominance attribute (0 or 1) to the feasible variants allows for dividing the feasible variants set into two subsets: (0) subset of dominated variants and (1) subset of nondominated variants (Fig. 3a). As can be seen, this information is quite general and does refer to the internal structure of the feasible objectives set, and in particular does not provide any knowledge about, e.g., the following: 1) what number of variants is dominated by each of feasible variants; 2) what number of solutions is dominated by a given nondominated variant; 3) how far away from Pareto front lies a given dominated feasible variant, etc. Goldberg (1989) proposed ordering the feasible solutions depending on the depth of consecutive fronts of nondominated objectives (Fig. 3b). As can be seen, the dominance attribute, which in this case is the domination depth, takes discrete values: 1, 2, 3, etc. More detailed analysis of the structure of feasible objectives set allows for the use of following concepts as dominance attributes: 1) rank of feasible variants, dominance rank (Fig. 3c) and 2) feasible variant evaluation dominance count (Fig. 3d). Values of both attributes vary in a continuous way over the set of feasible objectives, depending on the strategies adopted for their determination.

The dominance rank for a given variant is proportional to the number of feasible variants dominated by a given variant (Fig. 3c). The value of dominance rank is then highest for nondominated variants approximating the Pareto front. For the variants lying outside of Pareto front (or its approximation), deeper and deeper in the feasible objectives set, the values of dominance rank are a falling measure of their distance from the front. The dominance count of a feasible variant is proportional to the number of other feasible variants dominating a given variant (Fig. 3d). Its value is then the lowest and equals zero in case of nondominated variants, which approximate the Pareto front. For the variants lying outside of the Pareto front (or its approximation), deeper and deeper in the feasible variants set, the values of count are a rising measure of their distance from the front.

Other schemes for dominance attributes were proposed by, e.g., Fonseca and Fleming (1993) and Zitzler and Thiele (1999). Each method has its own particular feature, but the early experiences of the author and also other researchers, e.g., Leyland (2002), suggest that the influence of particular dominance rank strategies on the convergence of optimization process may be insignificant. In particular, while applying modern elitist strategies, the solutions quickly converge to  $s$  nondominated solutions set.

#### 5. Calculation tool for evolutionary multiobjective optimization of ship structure

##### 5.1. General

Genetic algorithms are an example of concepts of adaptation and evolutionary mechanism such as inheritance, mutation, natural

selection, and recombination (or crossover) application for solution of the complex optimization problems; see, e.g., Goldberg (1989), Michalewicz (1996), Coley (1999), and Man et al. (1999). They have already been extensively described in the literature, where the theoretical foundations, details of calculation procedures, and practical applications were discussed, e.g., Goldberg (1989), Davis (1991), and Michalewicz (1996); therefore, these problems are not addressed in this article.

The most important point of calculation tool for multiobjective optimization of ship structure is appropriate formulation of fitness function, which governs the optimization process. In the first step, a single, parameterized objective function was formulated, consisting of summing the optimization objectives with proper weight coefficients. The weight coefficients values of this substitute objective function are proposed by the expert corresponding to the multiobjective optimization strategy. The simplest concept is the introduction of objective function  $F(\mathbf{x})$  as a linear combination of  $S$  partial optimization objectives  $f_s(\mathbf{x})$ :

$$F(\mathbf{x}) = \sum_{s=1}^S w_s f_s(\mathbf{x}) \quad (5)$$

*substitute objective = optimization objectives*

where  $w_s$  are coefficients determining the weights assigned to particular optimization objectives and  $S$  is the number of objectives. The multiobjective optimization problem is in this way reduced to the single-objective problem.

Formulation of the fitness function in the form of substitute scalar optimization objective in form of equation (5) is a commonly accepted practice. In the case of a minimization problem, all partial optimization objectives,  $f_s(\mathbf{x}) \rightarrow \min!$ , substitute (aggregated) objective function is also minimized,  $F(\mathbf{x}) \rightarrow \min!$ . We note, however, that the genetic algorithms solve the problem of maximization of fitness function,  $\mathcal{F}(\mathbf{x}) \rightarrow \max!$ , which is a measure of quality of generated solutions and directly influences probability of selection of generated individuals (variants). As a result of this fact, to use the genetic algorithms, the minimization problem (5) must be transformed to the corresponding maximization problem. In the next part of this section, a proposition is presented and transformation of the optimization problem is discussed. It consists of the following stages: formulating utility functions of optimization objectives (Subsection 5.2), formulating a penalty function (Subsection 5.3), formulating a dominance rank (Subsection 5.4), formulating a dominance count (Subsection 5.5), formulating a distance to the asymptotic solution (Subsection 5.6), and formulating a combined fitness function (Subsection 5.7).

##### 5.2. Utility functions

Partial optimization objectives appearing in equation (5) were replaced by properly formulated utility functions of these objectives:  $f_s(\mathbf{x}) \rightarrow u_s(f_s[\mathbf{x}])$ :

$$u_s(\mathbf{x}) = \left( \frac{f_s(\mathbf{x})}{f_{s,max}} \right)^{r_s} \rightarrow \max! \Leftrightarrow f_s(\mathbf{x}) \rightarrow \max! \quad (6a)$$

$$u_s(\mathbf{x}) = \left( \frac{f_{s,max} - f_s(\mathbf{x})}{f_{s,max}} \right)^{r_s} \rightarrow \max! \Leftrightarrow f_s(\mathbf{x}) \rightarrow \min! \quad (6b)$$

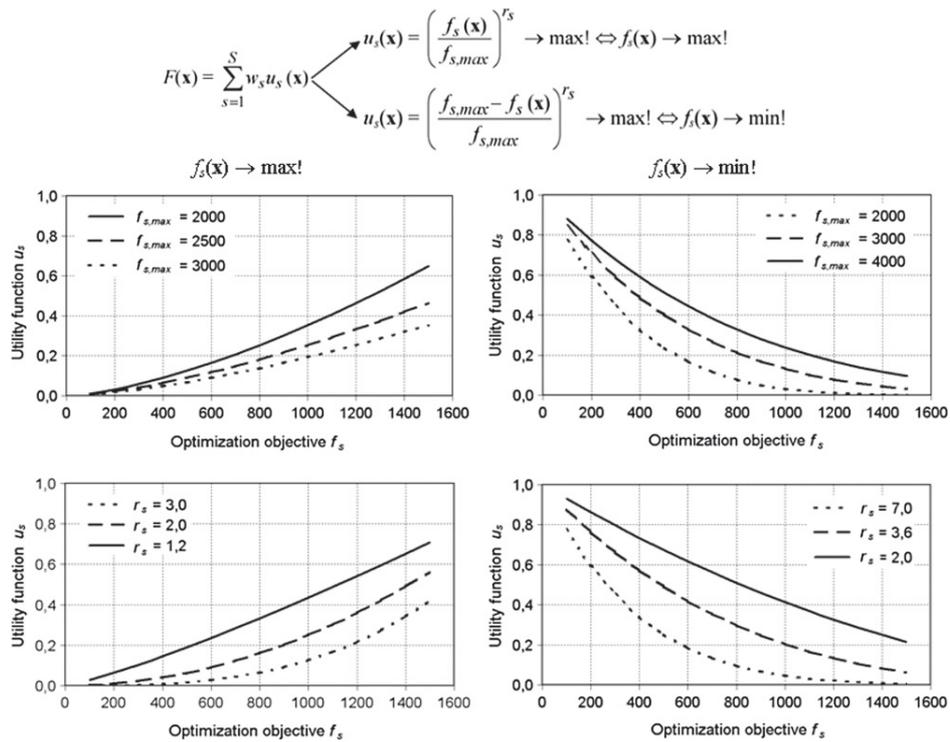


Fig. 4 Graphical illustration of the concept of utility function  $u_s(\mathbf{x})$  of optimization objectives  $f_s(\mathbf{x})$

where  $f_{s,max}$  are the greatest values of respective optimization objective anticipated in computations and  $r_s$  are positive exponents of respective utility functions. The values of the utility function are dimensionless and normalized to unity, that means  $u_s(f_s(\mathbf{x})) \rightarrow [0, 1]$ . Mathematical form of utility functions also assures that substitute scalar objective function is maximized for any types of optimization objectives; that means:

$$F'(\mathbf{x}) = \sum_{s=1}^S w_s u_s(\mathbf{x}) \rightarrow \max! \quad (7)$$

The appropriate values of  $f_{s,max}$  and  $r_s$  are selected by the user on the basis of the test calculations to achieve the required convergence of the algorithm. The distribution of the utility function for exemplary values of parameters  $f_{s,max}$  and  $r_s$  is presented in Fig. 4.

### 5.3. Penalty functions

A computer code for optimization of ship structures should allow for accounting for a series of design constraints such as the local and overall strength. On the other hand, implementation of the genetic algorithms requires that the equivalent problem is formulated without any constraints. Observing that the genetic algorithms do not require continuity nor the existence of derivative functions, a concept of the penalty function has been used by many researchers (Fox 1971; Ryan 1974; Reklaitis et al. 1983; Vanderplaats 1984). The augmented objective function of the

unconstrained maximization problem  $f(\mathbf{x})$  has been formulated as a penalty function:

$$f(\mathbf{x}) = \sum_{s=1}^S w_s u_s(\mathbf{x}) + \sum_{p=1}^P w_p P(\mathbf{x})_p^{r_p} \rightarrow \max! \quad (8)$$

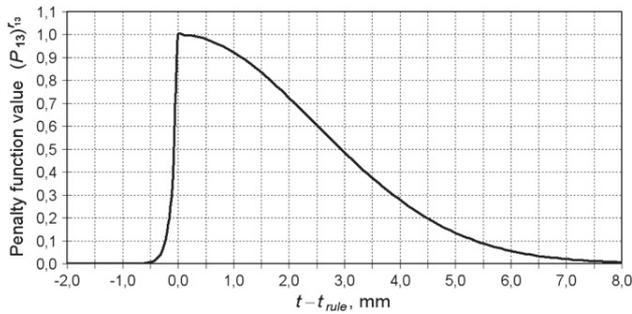
*augmented objective = objectives + constraints*

where:  $u_s(\mathbf{x})$  = utility function of objective function  $f_s(\mathbf{x})$ ,  $S$  = number of optimization objectives,  $P(\mathbf{x})_p$  = component of the penalty function for the violation of  $p$ -th constraint,  $w_p$  = penalty coefficient for the violation of  $p$ -th constraint,  $r_p$  = exponent of  $p$ -th component, and  $P$  = number of constraints.

The mathematical form of the exponential form of the penalty functions, the same for all the constraints, was formulated so that the penalty function values  $P(\mathbf{x})_p$  are dimensionless and normalized to unity. For example, in the case of rule requirement regarding main deck plate thickness, the penalty function takes the form:

$$P_{13}^{r_{13}} = \begin{cases} \left( e^{|t-t_{rule}|} \right)^{r_{13,I}} & \text{for } t < t_{rule} \\ \left( e^{(t_{rule}-t)^2} \right)^{r_{13,II}} & \text{for } t \geq t_{rule} \end{cases} \quad (9)$$

where:  $t$  = actual (generated by the algorithm) plate thickness and  $t_{rule}$  = rule thickness. The appropriate values of  $w_p$  as well as  $r_p$  are selected by the user on the basis of test calculations to achieve to desired convergence of the algorithm. Distribution of the penalty function for exemplary values of parameter  $r_p$  is presented in Fig. 5.

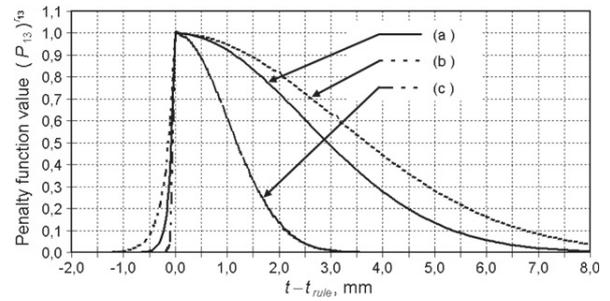


**Fig. 5** Graphical illustration of concept of penalty function;  $r_{13} = r_{13,I} = -10.0$  for  $t - t_{rule} < 0$ ,  $r_{13} = r_{13,II} = -0.08$  for  $t - t_{rule} \geq 0$

The form of the components of the penalty function taken for the analysis has a very interesting interpretation with respect to scantlings of the structural elements of a ship hull. It is shown in Fig. 5, for example, that values of the plate thickness,  $t$ , less than the values required by the rules  $t_{rule}$ ,  $t - t_{rule} < 0$ , are forbidden, because they do not meet the constraint. It follows that these values are not preferred (promoted, awarded) in the selection process. Individuals with the corresponding genes will be penalized with small values of the penalty function.<sup>c</sup> From the distribution of the penalty function, it follows that possibility of survival of solutions violating the strictly formulated constraints is admissible, although with a tiny probability. As a result of this fact, there is a possibility that the small number of solutions violating the constraints will be corrected in the next generations and will transmit other advantageous features to the descendants improving the algorithm convergence. In the process of selection, variants will be promoted of thickness slightly greater than required by the rules  $t_{rule}$ ,  $t - t_{rule} > 0$ . The greater values of the plating thickness,  $t$  will be less preferred because it is not necessary to increase the plate thickness excessively considering this constraint. Such individuals will be penalized for too large, despite their admissibility, values of the plate thickness  $t$ ; it is a small value of the corresponding component of the penalty function, which is the penalty in this case.<sup>d</sup> The adopted form of the penalty function can be interpreted similarly with respect to other parameters defining dimensions of the structural elements, e.g., section modulus, moment of inertia, cross-sectional area. Possibility to change the values of exponents  $r_p$  in the areas, that is  $r_{p,I}$  and  $r_{p,II}$ , as well as fine-fitting the shape of the penalty function (Fig. 6) causes it to be a very subtle and useful tool in controlling evolutionary optimization of structures.

<sup>c</sup>The proposed realization of selection causes that some individuals have a chance to participate in the reproduction, which slightly violate the imposed constraints but have other advantageous features. In the next generations, disadvantageous features can be removed or interchanged in the operations of mutation and crossing, and the changed variant can turn to be very advantageous solution. Such realization of selection increases the capability of searching the solution space and makes it possible to overcome “barriers” and “gulleys” in the multimodal solution space.

<sup>d</sup>Because so defined penalty function rewards good variants, it is natural to refer to it as a reward or preference function, rewarding, or preferring good variants, yet the common name penalty function is used as it is done in the references.



**Fig. 6** Illustration of influence of parameter  $r_p$  of penalty function on course of function (equation [9]): (a)  $r_{13} = r_{13,I} = -10.0$  for  $t - t_{rule} < 0$ ,  $r_{13} = r_{13,II} = -0.08$  for  $t - t_{rule} \geq 0$  (soft selection); (b)  $r_{13} = r_{13,I} = -5.0$  for  $t - t_{rule} < 0$ ,  $r_{13} = r_{13,II} = -0.05$  for  $t - t_{rule} \geq 0$  (dilution of soft selection); (c)  $r_{13} = r_{13,I} = -30.0$  for  $t - t_{rule} < 0$ ,  $r_{13} = r_{13,II} = -0.0$  for  $t - t_{rule} \geq 0$  (concentrate of soft selection)

As the augmented objective function  $f(\mathbf{x})$  expressed by the relation equation (8) with utility functions as well as penalty components dimensionless and normalized to unity is: 1) defined; 2) single-valued; and 3) ascending, having real values and positive in the search space, it can be adopted directly as the fitness function.

#### 5.4. Dominance rank

As we already know, the scheme of multiobjective optimization proposed in equation (8) allows only for rough differentiation of feasible solutions with regard to domination relation in Pareto sense (Fig. 3a) and does not account for information about how many solutions are dominated by a given solution.

For the solving of the mentioned problem, the author proposed a scheme in which the feasible solutions are ranked by the number of other solutions dominated by them relative to the number of feasible solutions in the current population. Therefore, dominance rank  $R_{fi}$  of  $i$ -th feasible solution is specified by an equation:

$$R_{fi}(i) = \frac{\sum_{j=1, j \neq i}^{N_{fi}} dm(i, j)}{N_{fi}} \rightarrow \max! \quad (10)$$

where  $dm(i, j) = 1$  when  $i$  dominates  $j$ , and  $dm(i, j) = 0$  in other cases,  $i, j =$  indices of verified feasible solutions,  $N_{fi} =$  number of feasible solutions in the current population (Fig. 3c). Advantages of the proposed strategy are: 1) ease of calculations; 2) standardization of dominance rank values in  $[0, 1]$  range; and 3) ascending values of dominance rank for solutions approaching the Pareto front (lying at the edge of feasible set). Thanks to properties (2) and (3), the value of dominance rank calculated in the proposed way may be directly included in the fitness function. In such a case, selection is going to promote feasible solutions located close to the Pareto front, whereas the solutions lying gradually further and further from the Pareto front are going to be promoted weaker and weaker, which is a numerical realization of selection pressure exerted on the solutions located close to the Pareto front and which thus enhances the exploitation performance of the

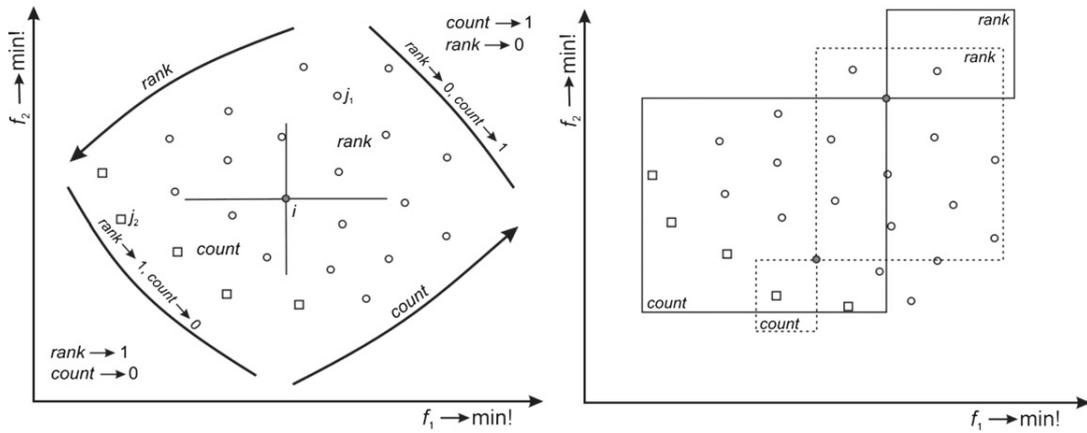


Fig. 7 Graphical illustration of the dominance rank and dominance count concepts;  $f_1 \rightarrow \min!$ ,  $f_2 \rightarrow \min!$

algorithm<sup>e</sup> (Fig. 7). A disadvantage of the proposed strategy is computational complexity  $N^2$ .

### 5.5. Dominance count

Similarly, feasible solutions can be classified by the number of solutions dominating them relative to the number of feasible solutions. Thus, evaluation dominance count  $C_{fi}$  of  $i$ -th feasible solution is expressed by the formula:

$$C_{fi}(i) = \frac{\sum_{j=1, j \neq i}^{N_{fi}} dm(j, i)}{N_{fi}} \rightarrow \max! \quad (11)$$

where  $dm(j, i) = 1$  when  $j$  dominates  $i$ , and  $dm(j, i) = 0$  otherwise,  $i, j =$  indices of verified feasible solutions, and  $N_{fi} =$  number of feasible solutions in the current population (Fig. 3d). The dominance count defined in this way has the mentioned properties (1) and (2) and property (3) ascending values of dominance count for the variants situated further and further from the Pareto front (located deep inside the feasible set). Thanks to properties (2) and (3), the value of dominance count calculated in the proposed way may also be directly included in the fitness function; in such a case, selection is going to promote feasible solutions located far from the Pareto front, whereas the solutions approaching the Pareto front are going to be promoted weaker and weaker, which is a numerical realization of selection pressure exerted on solutions located far from the Pareto front and which thus enhances the exploratory properties of the algorithm (Fig. 7). Similarly, like in the previous case, the disadvantage of the proposed strategy is computational complexity  $N^2$ .

As has already been mentioned, the strategies for dominance ranking and the dominance count of the feasible variants proposed by the author allow for their inclusion directly in the earlier formu-

lated (equation [8]) extended objective function of a unconstrained maximization problem  $f(\mathbf{x})$ :

$$f(\mathbf{x}) = \sum_{s=1}^S w_s u_s(\mathbf{x}) + w_{rank} R_{fi}(\mathbf{x}) + w_{count} C_{fi}(\mathbf{x}) + \sum_{p=1}^P w_p P(\mathbf{x})_p^{r_p} \rightarrow \max!$$

$$\begin{aligned} \text{combined objective} &= \text{optimization objectives} \\ &+ \text{dominance rank} + \text{dominance count} \\ &+ \text{constraints} \end{aligned} \quad (12)$$

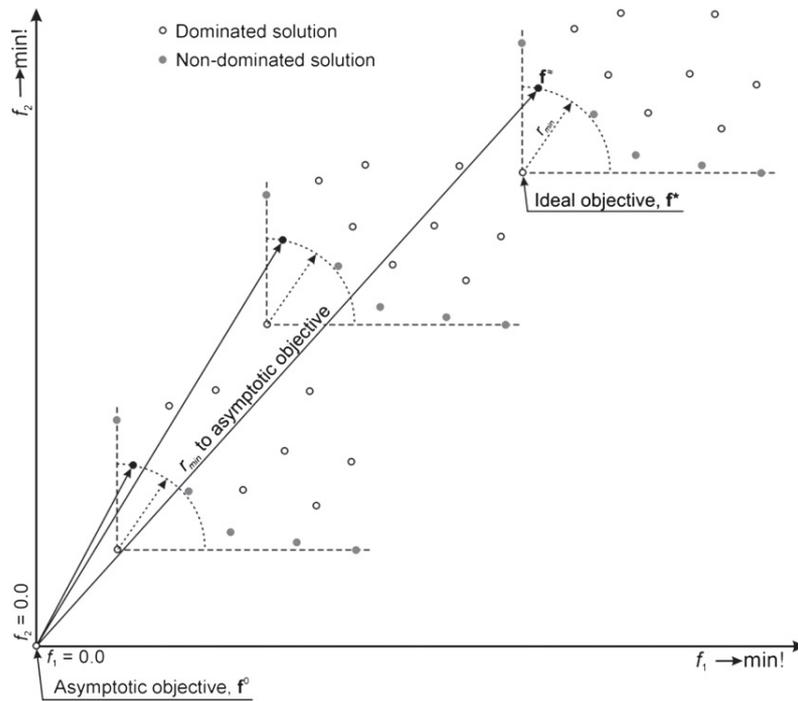
where:  $R_{fi}(\mathbf{x}) =$  dominance rank of feasible variant,  $w_{rank} =$  dominance rank weight coefficient,  $C_{fi}(\mathbf{x}) =$  dominance count of feasible variant, and  $w_{count} =$  dominance count weight coefficient. Assuming zero values of the weight coefficients,  $w_{rank}$  and  $w_{count}$ , the user can decide whether the corresponding domination attributes are on or off.

### 5.6. Distance to asymptotic solution

To indicate a single solution, which may be considered to be “the best” solution of a multiobjective optimization problem, and the monitoring of evolution of nondominated solution in the direction of theoretically lowest values of optimization objectives  $f_s \rightarrow 0!$  as moving the set of nondominated solutions in the desired direction, the author has then introduced a concept of a asymptotic objective (asymptotic solution), which represents an objective/solution corresponding to asymptotical values of optimization objectives:  $f_1 \rightarrow 0!$   $f_2 \rightarrow 0!$  (Fig. 8). With such a definition of an asymptotic solution, it is possible to: 1) determine the distance from each nondominated solution to this point and then choose an asymptotic closest solution, and additionally 2) monitor the

<sup>e</sup>Two strategies of gaining and using knowledge on the solution space are used in the evolutionary algorithms: exploitation—allowing for finding many advantageous solutions located in the vicinity of local optima, thus representing local efficiency of algorithm; and exploration—allowing for investigating a large part of the estimation space to localize/identify areas that can potentially contains advantageous solutions; in these regions, the local optimization algorithms can be then applied or increase exploitation properties of the algorithm.

<sup>f</sup>It is then assumed that in case of min!-type objectives the respective objectives (coordinates) are equal to zero. However, in case of max!-type objectives, the user shall set the value of these objectives (coordinates) as some known from experience values of respective objectives, which are impossible to attain, but which are going to be approached asymptotically by the nondominated solutions.



**Fig. 8** Graphical illustration of the movement of solutions nearest to the ideal solution  $f^*$  in the direction of asymptotic solution during the evolution, in case of min! type objectives; for constant distances from the ideal solution, the distance from asymptotic one changes (evolves) instead;  $f_1 \rightarrow \min!$ ,  $f_2 \rightarrow \min!$

evolution of set of nondominated solutions distance from this solution during the simulation.

The method of calculating the distances between feasible solutions and a asymptotic solution allows for including a distance from an asymptotic solution directly in the earlier formulated extended objective function used for unconstrained maximization problem,  $f(\mathbf{x})$ :

$$f(\mathbf{x}) = \sum_{s=1}^S w_s u_s(\mathbf{x}) + w_{rank} R_{fi}(\mathbf{x}) + w_{count} C_{fi}(\mathbf{x}) + w_{distance} [1 - d_{fi}(\mathbf{x})] + \sum_{p=1}^P w_p P(\mathbf{x})_p^r \rightarrow \max!$$

$$\begin{aligned} \text{combined objective} &= \text{objectives} + \text{dominance rank} \\ &+ \text{dominance count} + \text{distance to asymptotic solution} \\ &+ \text{constraints} \end{aligned} \quad (13)$$

where:  $d_{fi}(\mathbf{x})$  = distance between the feasible solution and the asymptotic solution and  $w_{distance}$  = distance weight coefficient.

### 5.7. Combined fitness function

Because the combined objective function  $F(\mathbf{x})$  expressed by equation (13) is: 1) well defined; 2) single-valued; and 3) ascending, having real values and positive in the search space, it has been adopted directly as the combined fitness function  $F(\mathbf{x})$ .

The combined fitness function used in the proposed form (equation [13]) includes instruments that provide for effective solving

of constrained multiobjective optimization ship structure problem. These are: *combined fitness* = *objectives* + *rank* + *count* + *distance* + *constraints*; *objectives* = represents selective pressure exerted in the direction of the desired values of optimization objectives; *dominance rank* as well as *dominance count* = represents selective pressure related to the location of a feasible variant with regard to nondominated objectives set, measured with appropriate dominance attributes; *distance* = represents selective pressure related to the distance between a feasible solution and the asymptotic solution; and *constraints* = represents the reduction of solution quality and the related reduction of selection probability caused by the violation of constraints. Because the values of dominance rank, dominance count, and the distance to asymptotic solution are calculated only for feasible variants, this involves also additional promotion of such variants. Proper use of the proposed components of the combined fitness function by the experienced user makes for a highly flexible and effective solving of multiobjective optimization problems based on a genetic algorithm or other evolutionary algorithms in general.

### 5.8. Computer code for multiobjective optimization of ship structures

The computer code used for multiobjective optimization of ship structures with combined fitness function incorporates specialized procedures for: 1) encoding the genotype and generation of the ship hull structure variants; 2) analysis of constraints; 3) analysis of feasible solutions set with regard to dominance relation; 4) control of domination, including the determination of domination attribute values, dominance rank and dominance count, and the building of

nondominated solutions set approximating the Pareto set; 5) calculations of the distance of feasible solutions to the asymptotic one; and 6) calculation of combined fitness function values while accounting for the domination attributes and distance to asymptotic solution of feasible solutions. A block-type diagram presenting the main concepts of developed calculation software is shown in Fig. 9.

In case of a multiobjective optimization problem, we have to consider how to collect and present the information about the determined nondominated solutions (Pareto-optimal) and how to archive them. It is commonly accepted to graphically present all the feasible solutions as points in the objective space. Only a part of them is going to be nondominated (dominating, Pareto-optimal) solutions, and their set is going to be called a nondominated set or a Pareto front containing tradeoff solutions (actually we know this is going to be a set of nondominated solutions approximating the Pareto set).

Nondominated solutions produced during the simulation are recorded in a separate set (file), which is continuously supplemented and updated during the simulation. The solutions collected in the nondominated solutions set may be dealt with in two possible ways: 1) set membership has no influence on the selection of individuals (egalitarian strategy); and 2) individuals from the set (nondominated solutions) enjoy a guaranteed participation in selection (elite strategy). Egalitarian strategy has been used in the underlying article.

It is a well-known fact that in case many optimization objectives are used, it is going to be impossible to find a single best solution, because such a solution does not exist. In practice, however, the user awaits automatic or quasautomatic determination of a single solution or a few solutions, which could be taken as a solution of the problem. Moreover, users are accustomed to

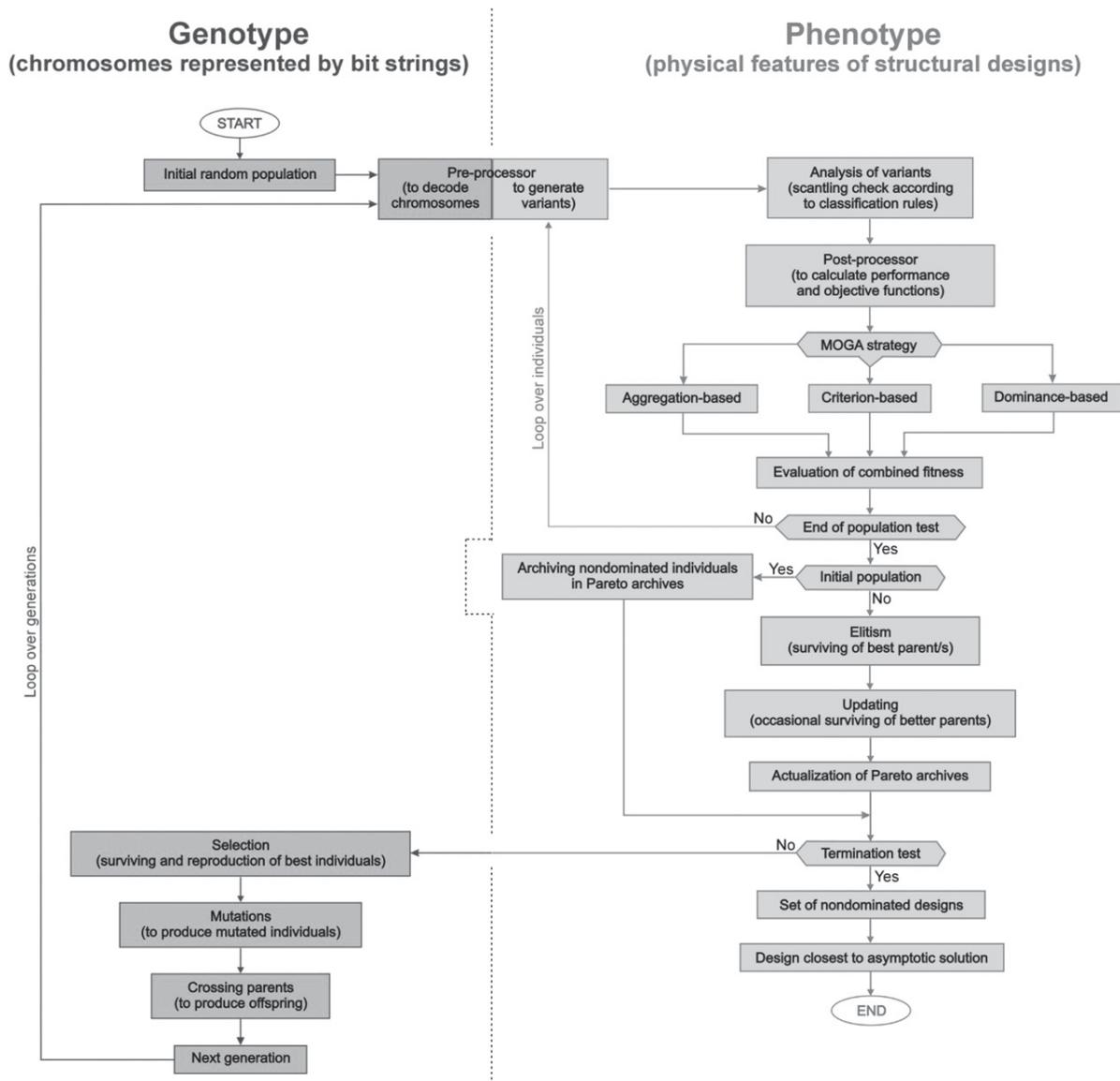


Fig. 9 Diagram presenting the main concepts of the developed computer software for the multiobjective optimization of seagoing ship structures based on a CFMOGA

the monitoring of evolution of a single value, which lets them evaluate the correctness of the calculation run, the convergence of solutions, and the quality of solutions being found. In single-objective cases, it is natural to monitor the values of fitness function and optimization objective. In case of multiobjective optimization, simultaneous evaluation of the evolving objectives is difficult to realize and interpret. To alleviate this problem, the author has used the concept of ideal or utopia solution, well known in the literature (Cohon 1978; Stadler 1988; Statnikov & Matosov 1995; Parsons & Scott 2004). In the generally accepted understanding, an ideal point refers to the lowest values of all objectives analyzed singly and not together. It means that if  $f_1^*, f_2^*, \dots, f_s^*, \dots, f_S^*$  will be used to denote the individual minima of each respective objective function, and the ideal solution (in objective space) is defined as  $\mathbf{f}^* = [f_1^* f_2^* \dots f_s^* \dots f_S^*]^T$ . Because  $\mathbf{f}^*$  simultaneously minimizes all objectives, it is an ideal solution that is rarely feasible. In such a case, however, it is then possible to locate a solution closest to the ideal point (nearest to the ideal solution):

$$\mathbf{f}_m^{\approx}(\mathbf{x}) = [f_{1,m}^{\approx}(\mathbf{x}) f_{2,m}^{\approx}(\mathbf{x})]^T \quad (14)$$

with the concept of closeness being understood here usually in the sense of Euclidean metrics in normalized, dimensionless objective space where  $\mathbf{f}_m^{\approx}(\mathbf{x})$  is the closest solution found in the  $m$  generation.

The concept of nondominated feasible solution nearest to the ideal objective  $\mathbf{f}^{\approx}(\mathbf{x})$  is sufficient to find a single solution, which may be considered to be “the best” solution of a multiobjective optimization problem. It is, however, inappropriate for the monitoring of evolution of nondominated solution in the direction of theoretically lowest values of optimization objectives  $f_s \rightarrow 0!$  because moving the set of nondominated solutions in the desired direction may also take place with unchanging distances of these solutions from an ideal solution (Fig. 8).

The author has then introduced earlier a concept of the asymptotic objective (asymptotic solution)  $\mathbf{f}^0$  (Fig. 8). According to this definition, the location of the ideal objective  $\mathbf{f}^*$  in the objective space is not fixed (Fig. 8), and this means that its location changes during simulation, so the distance to such a moving objective is not a good indicator of the solution quality. Therefore, a definition of asymptotic objective  $\mathbf{f}^0$  has been taken to make it stationary. It is then necessary to have such a definition, which would make the coordinates of this objective constant in the objective space, which means fixed values of optimization objectives. It is then assumed that in case of min!-type objectives, the respective objec-

tives (coordinates) are equal to zero. However, in case of max!-type objectives, the user shall set the value of these objectives (coordinates) according to his experience. These values should be unrealistic to attain, yet they are expected to be approached asymptotically by the nondominated solutions. After defining the asymptotic objective  $\mathbf{f}^0$ , let us assume that the nearest solution  $\mathbf{f}^{\approx}(\mathbf{x})$  is defined as the nearest solution with regard to the asymptotic objective  $\mathbf{f}^0$ .

## 6. Ship hull structural models for multiobjective optimization

### 6.1. General

Effectiveness of the present evolutionary algorithm for the multiobjective optimization of seagoing ship structures has been verified solving the multiobjective optimization problem for the midship segment of the passenger-car catamaran ferry based on the Auto Express 82 design developed by the Austal (HANSA 1997; Knaggs 1998). The following models were developed for the optimization: 1) ship structural model; 2) optimization model; and 3) genetic model.

### 6.2. Structural model of ship hull

Main particulars of the Austal Auto Express 82 vessel are given in Fig. 10. The general arrangement of the ship and her cross- and longitudinal sections is shown in Fig. 11. The cylindrical and prismatic model amidships—block-section ( $17.5 \times 23.0 \times 11.7$  m)—was taken, which is a simplification typical for the initial design. Boundaries of the block in the longitudinal direction are formed by the bulkheads. Nine structural regions can be distinguished in the model. All regions are longitudinally stiffened with stiffeners, their spacing being different in each structural region. The transverse web frame spacing is common for all the regions. Both types of spacing, stiffener and transverse frame, are considered as design variables. The transverse bulkheads are disregarded to minimize the number of design variables.

The structural material is aluminium alloy having properties given in Table 1. The 5083-H111 aluminium alloys are used for plates elements, whereas 6082-T6 aluminium alloys for bulb extrusions. The plate thicknesses and the bulb and T-bulb extruded stiffener sections are taken according to the commercial standards and given in Tables 2–4. Bulb extrusions are used as longitudinal stiffeners, whereas T-bulb extrusions are used as web frame profiles. Practically, the web frames are produced by welding the elements

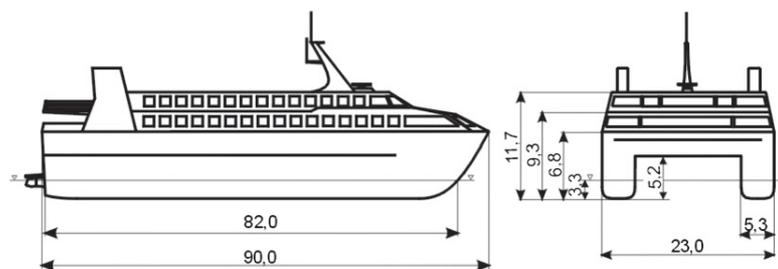


Fig. 10 High-speed vehicle-passenger catamaran, type Auto Express 82; main particulars (all dimensions are in meters)

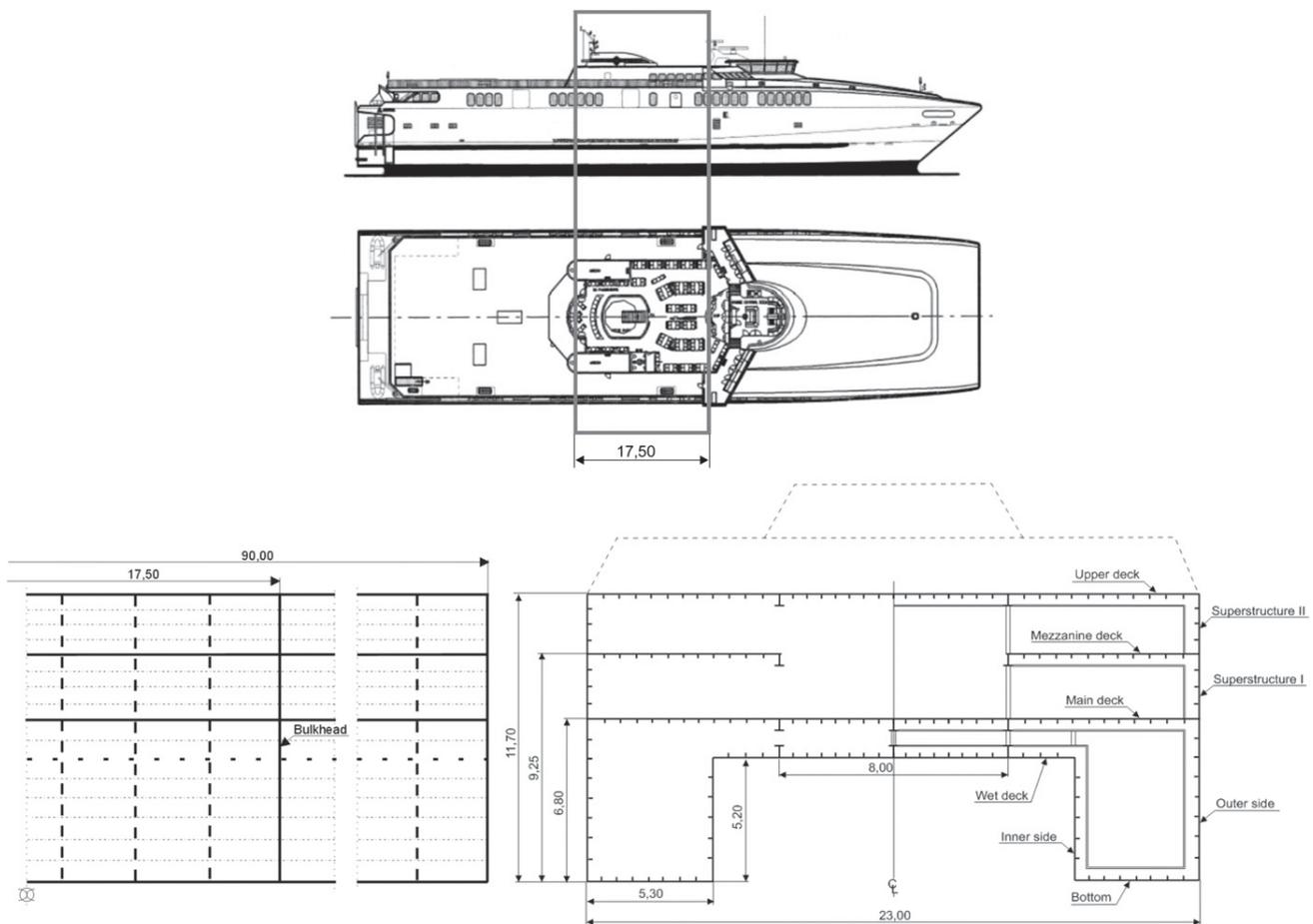


Fig. 11 Assumed model of craft—midship block-section, frame system, and structural regions (all dimensions are in meters)

cut out of the metal sheets. Dimensions of the prefabricated T-bar elements are described by the four following design variables: web height and thickness as well as flange breadth and thickness. In the case of extruded bulb, a single variable is sufficient to identify the profile, its dimensions, and geometric properties. It reduces the computational problem and accelerates analysis.

The strength criteria for calculation of plate thicknesses and section moduli of stiffeners and web frames are taken in accordance to the Bureau Veritas (1995) classification rules. It was assumed that bottom, wet deck, outer side, and superstructure I and II are subject to the pressure of water dependent on the speed

and navigation region. The main deck was loaded by weight of the trucks transmitted through the tires, mezzanine deck—weight of the cars while the upper deck—the weight of equipment and passengers. Pressures were calculated according to the procedures of the classification rules.

Structural loading was assumed corresponding to the specific loading case. The loading case should be the one defining the most severe requirements for the structural dimensions. Because this is not certain, it can be assumed that there are more severe loading

Table 2 Thickness of plates

No.	Thickness $t$ (mm)	No.	Thickness $t$ (mm)
1	3.00	8	12.00
2	4.00	9	15.00
3	5.00	10	20.00
4	6.00	11	30.00
5	7.00	12	40.00
6	8.00	13	50.00
7	10.00	14	60.00

Table 1 Properties of structural material—aluminum alloys

No.	Property	Value
1	Yield stress $R_{0.2}$	125 (for 5083-H111 alloy) N/mm <sup>2</sup> 250 (for 6082-T6 alloy) N/mm <sup>2</sup>
2	Young modulus $E$	70,000 N/mm <sup>2</sup>
3	Poisson ratio $\nu$	0.33
4	Density $\rho$	26.1 kN/m <sup>3</sup>

**Table 3 Dimensions of bulb extrusions**

No.	Dimensions ( $h, b, s, s_1$ )* (mm)	Cross-sectional area (cm <sup>2</sup> )
1	80 × 19 × 5 × 7.5	5.05
2	100 × 20.5 × 5 × 7.5	6.16
3	120 × 25 × 8 × 12	11.64
4	140 × 27 × 8 × 12	13.64
5	150 × 25 × 6 × 9	10.71
6	160 × 29 × 7 × 10.5	13.51
7	200 × 38 × 10 × 15	24.20

\* $h$ , cross-section height;  $b$ , flange width;  $s$ , web thickness;  $s_1$ , flange thickness.

**Table 4 Dimensions of T-bulb extrusions (table contains only profiles having largest dimensions, selected by author for the analysis out of total number of 52)**

No.	Sizes ( $h, b, s, s_1$ )* (mm)	Cross-sectional area (cm <sup>2</sup> )
42	400 × 140 × 5 × 8	30.80
43	410 × 100 × 6 × 8	32.12
44	420 × 150 × 5 × 10	35.50
45	420 × 150 × 8 × 10	47.80
46	450 × 100 × 9 × 10	49.60
47	450 × 150 × 10 × 12	61.80
48	500 × 100 × 10 × 12	62.00
49	500 × 150 × 10 × 12	68.00
50	500 × 200 × 12 × 14	88.00
51	500 × 250 × 12 × 14	95.00
52	550 × 250 × 14 × 16	117.00

\* $h$ , cross-section height;  $b$ , flange width;  $s$ , web thickness;  $s_1$ , flange thickness.

cases and the results of optimization for these cases can change the values of the optimization objectives, in the disadvantageous direction. This can, for example, increase the structural weight.

The minimum structural weight (volume of structure) and outer area of structural elements for maintenance (cleaning, painting, etc.) were taken as optimization objectives in the study and were introduced in the definition of the substitute scalar objective function and constraints based on the classification rules. Where structural weight and surface area are chosen as the objective functions their values depend only on the geometrical properties of the structure (if structural material is fixed). The assumed optimization task is simple but the main objective of the study was to build the computational method, verify the computer code and prove its application for the multiobjective optimization of a ship hull.

### 6.3. Multiobjective optimization model of ship hull structure

The most general formulation to solve the ship structural multiobjective optimization problem is defined as finding a combination of values of the vector of design variables  $\mathbf{x} = [x_1 \dots x_i \dots x_n]^T$  defining the structure, which optimizes vector objective function  $\mathbf{f}(\mathbf{x})$ . The design variables should also meet a complex set of constraints imposed on their values. The constraints formulate the set of feasible solutions. It is assumed that all functions of the multiobjective optimization problem are real and the number of constraints is finite.

For the multiobjective optimization problem, an aggregated objective function can be formulated, with the use of the components of the vector objective function, in the following form:

$$F(\mathbf{x}) = F(f_1(\mathbf{x}), f_2(\mathbf{x})) = w_1 f_1(\mathbf{x}) + w_2 f_2(\mathbf{x}) \rightarrow \min! \quad (15)$$

where  $f_1(\mathbf{x})$  is a structural weight of midship block-section taken for optimization,  $f_2(\mathbf{x})$  is an area of the outer surface of structural members subjected to cleaning and painting operations (surface area for maintenance) in the section, and  $w_1$  and  $w_2$  are weight coefficients used for optimization objectives. Both optimization objectives influence the building and operational costs. Maintaining the costs at the lowest possible level increases the economical efficiency of investment. Weight coefficient values enable considering specific stakeholder preferences in the optimization.

Using the concept of an utility function, described in Subsection 5.2 and taking into consideration, in the form of a penalty function described in Subsection 5.3, the operational loads as well as the constraints imposed on the design variables, especially resulting from conditions of local and global strength formulated in the approved rules, the aggregated objective function can be expressed as the augmented objective function of the unconstrained maximization problem:

$$f(\mathbf{x}) = w_1 u_1(\mathbf{x}) + w_2 u_2(\mathbf{x}) + \sum_{p=1}^P w_p P(\mathbf{x})_p^{r_p} \rightarrow \max! \quad (16)$$

where all symbols are described before.

The augmented objective function (equation [16]) has been extended by the components corresponding to the dominance attributes (dominance rank and dominance count described in Subsection 5.4 and Subsection 5.5, respectively) and distance to the asymptotic solution (described in Subsection 5.6). As a consequence, the following form of the combined objective function has been derived:

$$f(\mathbf{x}) = w_1 u_1(\mathbf{x}) + w_2 u_2(\mathbf{x}) + w_{rank} R_{fi}(\mathbf{x}) + w_{count} C_{fi}(\mathbf{x}) + w_{distance} [1 - d_{fi}(\mathbf{x})] + \sum_{p=1}^P w_p P(\mathbf{x})_p^{r_p} \rightarrow \max! \quad (17)$$

Also in this case, all symbols are as described before.

As it has already been stated earlier, three aggregation-based multiobjective evolutionary strategies for taking account of the partial optimization objectives  $f_1(\mathbf{x})$  and  $f_2(\mathbf{x})$  are used in the scalar objective function (equation [15]) calculation and therefore also in the fitness function value calculation (equation [17]):

- selection of variants using the scalar objective function with the values of weight coefficients  $w_1$  and  $w_2$  set by the user ( $w\_strategy = 2$ );
- selection of variants using the scalar objective function with the values of weight coefficients  $w_1$  and  $w_2$  randomly and independently generated in the range [0, 1] ( $w\_strategy = 4$ ); and
- selection of variants using the random selected single partial optimization objective  $F(\mathbf{x}) = w_1 f_1(\mathbf{x})$  or  $F(\mathbf{x}) = w_2 f_2(\mathbf{x})$  ( $w\_strategy = 3$ ), which is implemented by the random selection of a single nonzero value of weight coefficient of the corresponding objective.

Additionally, it is also possible to have (equation [17]):

- selection of variants without ( $w_{rank} = 0$ ) or with ( $w_{rank} \neq 0$ ) taking into account the dominance rank of feasible solutions;
- selection of variants without ( $w_{count} = 0$ ) or with ( $w_{count} \neq 0$ ) taking into account the dominance count of feasible solutions; and
- selection of variants without ( $w_{distance} = 0$ ) or with ( $w_{distance} \neq 0$ ) taking into account the distance of feasible solutions to the asymptotic solution.

In the present formulation, a set of 37 design variables is applied (Table 5; Fig. 12). Introduction of a design variable representing the number of transversal frames in the considered section:  $x_4$ , and numbers of longitudinal stiffeners in the regions:  $x_5, x_9, x_{13}, x_{17}, x_{21}, x_{25}, x_{29}, x_{33}, x_{37}$  enables simultaneous optimization of both topology and scantlings within the single ship structural optimization model.

Numbers of stiffeners and transverse web frames, varying throughout the processes of optimization, determine corresponding spacing. Scantlings and weights of structural elements: plating, stiffeners, and frames are directly dependent on the stiffeners and frames spacing, topological properties of the structure.

The behavior constraints, ensuring that the designed structure is on the safe side, were formulated for each region according to the Bureau Veritas (1995) classification rules constituting a part of set of inequality constraints  $g_i(\mathbf{x})$ . The procedure for formulating the mathematical form of the penalty function considering the behavior constraints is presented in Subsection 5.3. After conducting series of test computations, an identical mathematical exponential form of penalty function was accepted for all constraints. Weight coefficients,  $w_p$ , allow for implementing comparative, in relation to the others, meaning of a given constraint identified by index  $p$ . Increasing the value of a weight

**Table 5 Simplified specification of bit representation of design variables**

No. $i$	Symbol $x_i$	Item	Substring length (no. of bits)	Value	
				Lower limit $x_{i,min}$	Upper limit $x_{i,max}$
1	$x_1$	Serial no. of mezzanine deck plate	4	1	10
2	$x_2$	Serial no. of mezzanine deck bulb	3	1	7
3	$x_3$	Serial no. of mezzanine deck T-bulb	4	42	52
4	$x_4$	Number of web frames	3	10	16
5	$x_5$	Number of mezzanine deck stiffeners	4	25	40
6	$x_6$	Serial no. of superstructure I plate	4	1	10
7	$x_7$	Serial no. of superstructure I bulb	3	1	7
8	$x_8$	Serial no. of superstructure I T-bulb	4	42	52
9	$x_9$	Number of superstructure I stiffeners	3	4	11
10	$x_{10}$	Serial no. of inner side plate	4	1	10
11	$x_{11}$	Serial no. of inner side bulb	3	1	7
12	$x_{12}$	Serial no. of inner side T-bulb	4	42	52
13	$x_{13}$	Number of inner side stiffeners	3	18	25
14	$x_{14}$	Serial no. of bottom plate	4	1	12
15	$x_{15}$	Serial no. of bottom bulb	3	1	7
16	$x_{16}$	Serial no. of bottom T-bulb	4	42	52
17	$x_{17}$	Number of bottom stiffeners	4	15	25
18	$x_{18}$	Serial no. of outer side plate	4	1	12
19	$x_{19}$	Serial no. of outer side bulb	3	1	7
20	$x_{20}$	Serial no. of outer side T-bulb	4	42	52
21	$x_{21}$	Number of outer side stiffeners	4	18	33
22	$x_{22}$	Serial no. of wet deck plate	4	1	12
23	$x_{23}$	Serial no. of wet deck bulb	3	1	7
24	$x_{24}$	Serial no. of wet deck T-bulb	4	42	52
25	$x_{25}$	Number of wet deck stiffeners	4	25	40
26	$x_{26}$	Serial no. of main deck plate	4	2	12
27	$x_{27}$	Serial no. of main deck bulb	3	1	7
28	$x_{28}$	Serial no. of main deck T-bulb	4	42	52
29	$x_{29}$	Number of main deck stiffeners	4	25	40
30	$x_{30}$	Serial no. of superstructure II plate	4	1	10
31	$x_{31}$	Serial no. of superstructure II bulb	3	1	7
32	$x_{32}$	Serial no. of superstructure II T-bulb	4	42	52
33	$x_{33}$	Number of superstructure II stiffeners	3	4	11
34	$x_{34}$	Serial no. of upper deck plate	4	1	10
35	$x_{35}$	Serial no. of upper deck bulb	3	1	7
36	$x_{36}$	Serial no. of upper deck T-bulb	4	42	52
37	$x_{37}$	Number of upper deck stiffeners	4	25	40
		Multivariable string length (chromosome length)	135		

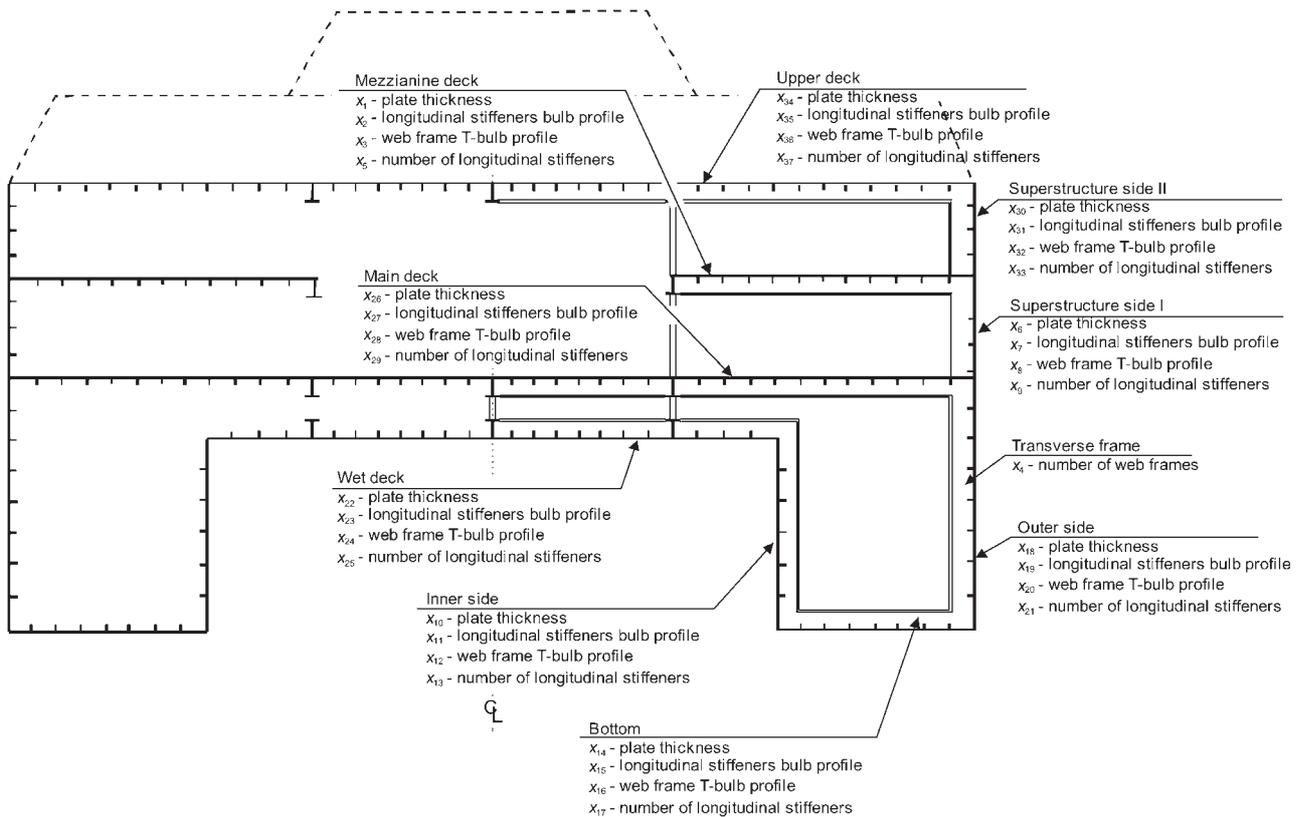


Fig. 12 Assumed model of craft—specification of design variables

coefficient will cause selective pressure increase from the item corresponding to the weight coefficient.

Side constraints, mathematically defined as inequality constraints, for design variables are given in Table 5. They correspond to the limitations of the range of the profile set. Some of them are formulated according to the author's experience in improving the calculation convergence.

#### 6.4. Genetic model of the ship hull structure

**6.4.1. Chromosome structure.** The space of possible solutions is a space of structural variants of the assumed model. The ship hull structure model is identified by the vector  $\mathbf{x}$  of 37 design variables,  $x_i$ . Each variable is represented by a string of bits used as chromosome substring in Genetic Algorithm (GA). The simple binary code was applied. Such coding implies that each variant of solution is represented by a bit string named chromosome. The length of a chromosome, which represents structural variant, is equal to the sum of all substrings. The number of possible solutions equals the product of values of all variables. In this article, the ship structure coded in chromosome in which the length is equal to 135 bits making the number of possible solutions equal approximately  $10^{40}$ .

**6.4.2. Fitness function.** A fitness function is used to determine how the ship structure is suitable to meet the design conditions regarding GA. As was earlier explained, because combined objective function  $f(\mathbf{x})$  expressed by equation (17) fulfills requirements

of the fitness function, it has been adopted directly as combined fitness function  $F(\mathbf{x})$ . For two optimization objectives,  $f_1(\mathbf{x})$  and  $f_2(\mathbf{x})$ , the combined fitness function takes the form:

$$F(\mathbf{x}) = w_1 u_1(\mathbf{x}) + w_2 u_2(\mathbf{x}) + w_{rank} R_{fi}(\mathbf{x}) + w_{count} C_{fi}(\mathbf{x}) + w_{distance} [1 - d_{fi}(\mathbf{x})] + \sum_{p=1}^P w_p P(\mathbf{x})_p^r \rightarrow \max! \quad (18)$$

*combined fitness = optimization objectives + dominance rank + dominance count + distance to asymptotic solution + constraints*

**6.4.3. Genetic operators.** The basic genetic algorithm (*Simple Genetic Algorithm [SGA]*) produces variants of the new population using three main operators that constitute the GA search mechanism: selection, mutation, and crossover. The present algorithm is extended by introduction of elitism and updating.

Many authors described the selection operators, which are responsible for chromosome selection resulting from the value of their fitness function (Goldberg & Deb 1991; De Jong 1995; Back 1996; Michalewicz 1996). After the analysis of the selection operators, a roulette concept was applied for the proportional selection. The roulette wheel selection is a process in which individual chromosomes (strings) are chosen according to

their fitness function values; it means that strings with higher fitness values have higher probability of reproducing new strings in the next generation.

The mutation operator, which introduces random changes of the chromosome, was also described (Back 1996; Michalewicz 1996). Mutation is a random modification of the chromosome. It gives new information to the population and adds diversity to the mate pool (pool of parents selected for reproduction). Without the mutation, it is hard to reach a solution point, which is located far from the current population, whereas as a result of introduction of the random mutation operator, the probability of reaching any point in the search space never equals zero. This operator also prevents the premature convergence of GA to one of the local optima solutions, thus supporting exploration of the global search space. The simple bit mutation with probability  $p_m$  was applied.

The crossover operator combines the features of two parent chromosomes to create new solutions. The crossover allows exploitation of a local area in the objective space. Analysis of the features of the described operators (Goldberg & Deb 1991; Back 1996; Michalewicz 1996) led to developing a new,  $n$ -point, random crossover operator. The crossover parameters in this case are: the lowest  $n\_x\_site\_min$  and the greatest  $n\_x\_site\_max$  number of the crossover points and the crossover probability  $p_c$ . The operator works automatically and independently for each pair being intersected (with probability  $p_c$ ), and it sets the number of crossover points  $n\_x\_site$ . The number of points is a random variable inside the set range  $[n\_x\_site\_min, n\_x\_site\_max]$ . The test calculations proved high effectiveness and quicker convergence of the algorithm in comparison to algorithm realizing single-point crossover. It was also found that the number of crossover points  $n\_x\_site\_max$  greater than seven did not improve convergence of the algorithm. Therefore, the lowest and greatest values of the crossover points were set as following:  $n\_x\_site\_min = 1, n\_x\_site\_max = 7$ .

The effectiveness of the algorithm was improved applying an additional updating operator as well as introducing the elitist strategy.

Random character of selection, mutation, and crossing operators can have the effect that these are not the best fitting variants of the parental population, which will be selected for crossing. Even in the case they will be selected, the result will be that progeny may have less adaptation level. Thus, the efficient genome can be lost. The elitist strategy mitigates the potential effects of loss of the genetic material copying certain number of the best adapted parental individuals to the progeny generation. In most cases, the elitist strategy increases the rate of dominating population by well-adapted individuals, accelerating the convergence of the algorithm. The algorithm selects a fixed number of parental individuals,  $n_p$ , having the greatest values of the fitness function and the same number of descendant individuals having the least values of the fitness. Selected descendants are substituted by the selected parents. In this way, the operator increases the convergence in the objective space. An update operator with fixed probability of updating  $p_u$  introduces an individual, randomly selected from the parental population, to the progeny population, replacing a descendant less-adapted individual. This operator enhances exploration of objective space at the cost of decreasing the search convergence. It also prevents the algorithm from converging to a local minimum. These operators act in the opposite

directions, and they should be well balanced: exploitation of attractive areas found in the objective space as well as exploration of the objective space to find other attractive areas depending on the user's experience.

**6.4.4. Control parameters.** A single program run with the defined genetic model is characterized by values of 18 control parameters. In this case, the set of genetic model parameters set for each simulation run signed as *symi* includes 18 elements:

$$\text{symi} = (n_{dv}, l_{ch}, n_g, n_i, n_p, p_m, \\ p_c, c\_strategy, n\_x\_site\_min, n\_x\_site\_max, p_u, elitist, \\ w\_strategy, w_1, w_2, w_{rank}, w_{count}, w_{distance})$$

where  $n_{dv}$  = number of design variables (number of genes),  $l_{ch}$  = chromosome length (number of bits),  $n_g$  = number of generations,  $n_i$  = size of population,  $n_p$  = number of pretenders<sup>g</sup>,  $p_m$  = mutation probability,  $p_c$  = crossover probability,  $c\_strategy$  = denotation of crossover strategy (0 for fixed, 1 for random number of crossover points),  $n\_x\_site\_min$  = the lowest number of crossover points,  $n\_x\_site\_max$  = the greatest number of crossover points,  $p_u$  = update probability, *elitist* = logical variable to switch on (*elitism* = *yes*) and off (*elitism* = *no*) the pretender selection strategy,  $w\_strategy$  = denotation of strategy for aggregation of the substitute scalar objective function<sup>h</sup>,  $w_1$  = weight coefficient for the weight of structure,  $w_2$  = weight coefficient of surface area of the structural element for cleaning and painting,  $w_{rank}$  = weight coefficient of the solution dominance rank,  $w_{count}$  = weight coefficient of the individual dominance count, and  $w_{distance}$  = weight coefficient of the distance of individual from the asymptotical solution. These 18 parameters control the successive simulation runs and identify them uniquely for the adopted structure model.

For selection of more control parameters, it is not possible to formulate quantitative premises because of the lack of an appropriate mathematical model for the analysis of the GA convergence in relation to control parameters. The control parameters were set as a result of test calculations results to achieve a required algorithm convergence; their values are presented in Table 6.

## 7. Computational investigation—search for a set of nondominated solutions

### 7.1. Computational investigations program

To verify the suitability of the proposed method and the computer code developed for seeking of Pareto-optimal solutions of the

<sup>g</sup>In the case the elitist selection strategies, it means the challenger: selected from the population of parental individual who can replace, under certain conditions, the selected individual from the offspring population.

<sup>h</sup> $w\_strategy = 1$ : fixed value of weight coefficients  $w_1 = 0.0, w_2 = 0.0$  (switch off all optimization objectives);

$w\_strategy = 2$ : fixed value of weight coefficients  $w_1 = 0.5, w_2 = 0.5$  (switch on all optimization objectives);

$w\_strategy = 3$ : random value of weight coefficients  $w_1 = \text{random } 0 \text{ or } 1, w_2 = \text{random } 0 \text{ or } 1$  (random selection of single optimization objectives); and

$w\_strategy = 4$ : random value of weight coefficients  $w_1 = \text{random in } [0, 1], w_2 = \text{random in } [0, 1]$  (random contribution of all optimization objectives).

**Table 6 Control parameters of computational investigations**

No.	Designation sym <i>i</i>	Specification ( <i>n<sub>d</sub></i> , <i>l<sub>ch</sub></i> , <i>n<sub>g</sub></i> , <i>n<sub>i</sub></i> , <i>n<sub>p</sub></i> , <i>p<sub>m</sub></i> , <i>p<sub>c</sub></i> , <i>c_strategy</i> , <i>n_x_site_min</i> , <i>n_x_site_max</i> , <i>p<sub>it</sub></i> , <i>elitist</i> , <i>w_strategy</i> , <i>w<sub>1</sub></i> , <i>w<sub>2</sub></i> , <i>w_rank</i> , <i>w_count</i> , <i>w_distance</i> )
Series 1		
1	sym1-1	(37, 135, 10,000, 5,000, 10, 0.086, 0.800, 1, 1, 7, 0.33, <i>yes</i> , 2, 0.5, 0.5, 0.0, 0.0, 0.0)
2	sym1-2	(37, 135, 10,000, 5,000, 10, 0.086, 0.800, 1, 1, 7, 0.33, <i>yes</i> , 4, random in [0,1], random in [0,1], 0.0, 0.0, 0.0)
3	sym1-3	(37, 135, 10,000, 5,000, 10, 0.086, 0.800, 1, 1, 7, 0.33, <i>yes</i> , 3, random 0 or 1, random 0 or 1, 0.0, 0.0, 0.0)
Series 2		
4	sym2-1	(37, 135, 10,000, 5,000, 10, 0.086, 0.800, 1, 1, 7, 0.33, <i>yes</i> , 1, 0.0, 0.0, 3.0, 0.0, 0.0)
5	sym2-2	(37, 135, 10,000, 5,000, 10, 0.086, 0.800, 1, 1, 7, 0.33, <i>yes</i> , 1, 0.0, 0.0, 0.0, 3.0, 0.0)
6	sym2-3	(37, 135, 10,000, 5,000, 10, 0.086, 0.800, 1, 1, 7, 0.33, <i>yes</i> , 1, 0.0, 0.0, 0.0, 0.0, 3.0)

formulated multiobjective seagoing ship structure optimization problem, a number of calculation experiments has been carried out (Table 6) using the ship structural models earlier formulated and discussed in Section 5.

The purpose of the simulation was searching nondominated variants with respect to two optimization objectives with varying strategies for setting the values of weight coefficients for various objectives as well as dominance attributes:

- (1) Series 1: simulations marked with symbols sym1-1, sym1-2 and sym1-3.

In the simulation marked as **sym1-1**, fixed values of weight coefficients are used for whole simulation:  $w_1 = 0.5$  and  $w_2 = 0.5$ , which refer to the classic method of weighted objectives. In the simulation marked as **sym1-2**, the values of weight coefficients,  $w_1$  and  $w_2$ , were generated by the computer code as **random variables in the range [0, 1]**, which was done independently for each variant whenever the value of fitness function is calculated. In the simulation marked as **sym1-3**, the values of weight coefficients  $w_1$  and  $w_2$  were generated by the computer code as **random variables equal to either 0 or 1**, which was done independently for each variant whenever the value of fitness function is calculated; the value of 1 was used only for one, randomly selected objectives with the remaining ones equal to 0.

- (2) Series 2: simulations marked with symbols sym2-1, sym2-2 and sym2-3.

Search for nondominated variants while excluding the optimization objectives from the process of variant selection  $w_1 = w_2 = 0.0$  ( $w\_strategy = 1$ ), which was governed in particular simulations only by: 1) the value of the dominance rank of feasible solution,  $w\_rank = 3.0$ ,  $w\_count = 0.0$ ,  $w\_distance = 0.0$ , in the simulation marked as **sym2-1**; 2) the value of feasible variant dominance count,  $w\_count = 3.0$ ,  $w\_rank = 0.0$ ,  $w\_distance = 0.0$ , in the simulation marked as **sym2-2**; and 3) the distance between the feasible variant and the asymptotic solution,  $w\_distance = 3.0$ ,  $w\_rank = 0.0$ ,  $w\_count = 0.0$  in the simulation marked as **sym2-3**. The purpose of this simulation series was to find out whether the developed tool is effective in case of evolution being governed only by 1) dominance rank; 2) dominance count; or 3) distance from a asymptotic solution. This refers to the modern algorithms of the evolutionary multiobjective optimization, where the evolution is governed only by the dominance attributes.

In all simulations, the functions of penalties imposed for the violations of constraints were active,  $w_p \neq 0, p = 1, 2, \dots, P$ .

The computational investigations were carried out for two-objective problems, like in this case, it is possible to present obtained results graphically in a multitude of ways, which facilitate their interpretation and analysis.

### 7.2. Results of computational investigations

The change of the macroscopic values characterizing the evolution of the population of the ship structural solutions generated during the simulations is presented in Fig. 13: 1) the greatest fitness function value  $f_{max}$  and 2) the lowest distance from a feasible solution to the asymptotic solution. Figure 14 and Fig. 15 present a structure of the nondominated solutions sets of the last generation. For each nondominated solutions set, the variant closest to the asymptotic solution was specified.

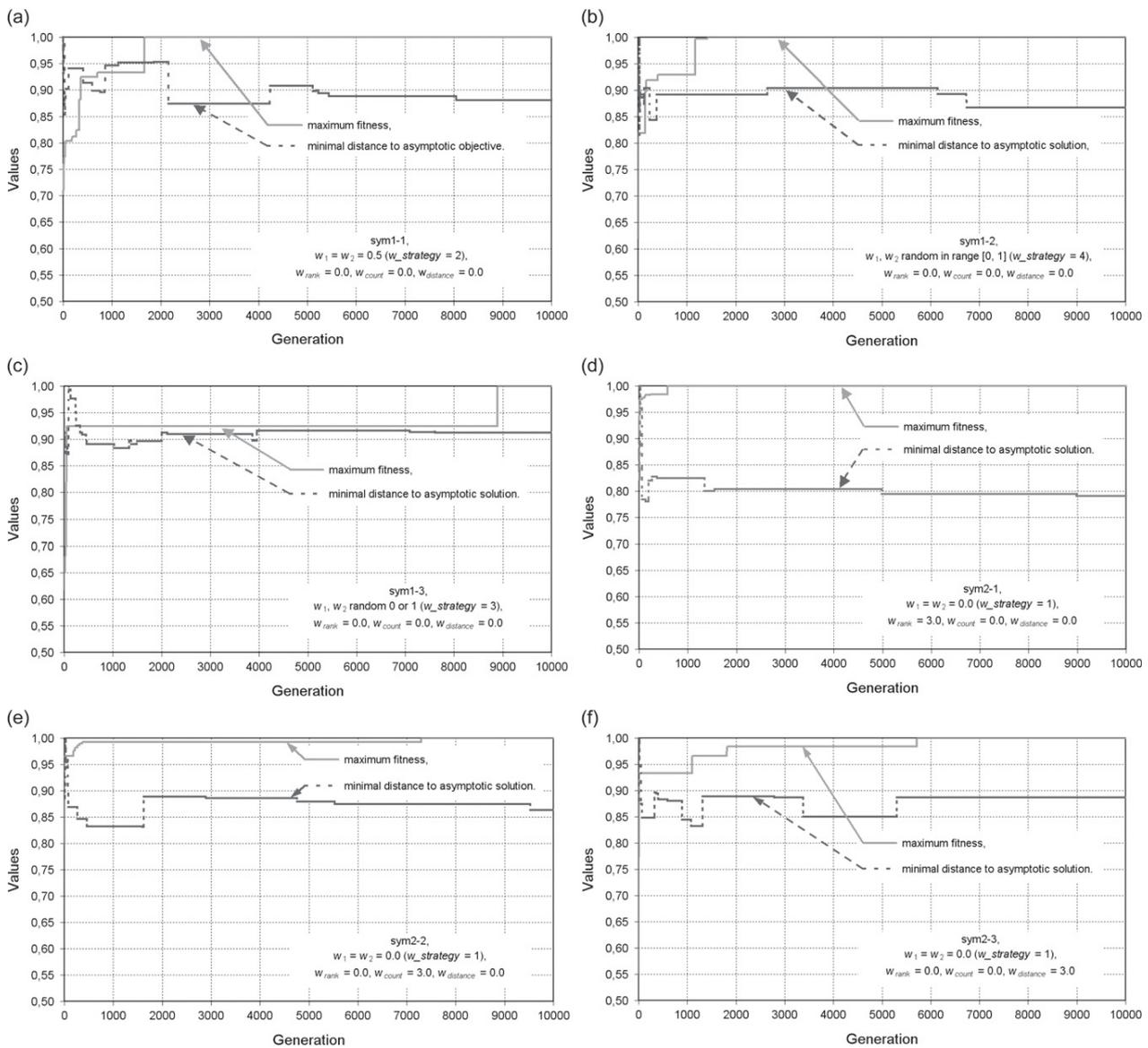
For example, in **sym1-1**, the variant closest to the asymptotic solution, which was found in 5116 generation, is characterized by: the distance of 1.096 in the normalized objective space to the asymptotic solution, the structural weight 1086.28 kN, and the surface area of structural elements 7422.11 m<sup>2</sup>. This variant may be recommended if a single solution for the formulated ship structural multiobjective optimization problem is to be selected:

$$\begin{aligned} \mathbf{f}_{\text{sym1-1}}^{\approx} &= \mathbf{f}_{5116}^{\approx}(\mathbf{x}) = \left[ f_{1,5116}^{\approx}(\mathbf{x}) f_{2,5116}^{\approx}(\mathbf{x}) \right]^T \\ &= [1086.28 \text{ kN } 7,422.11 \text{ m}^2]^T. \end{aligned}$$

### 7.3. Analysis of results and the conclusions obtained from the computational simulations

Two series of the computer simulations confirmed effectiveness of the developed computational algorithm and computer code for solution of the formulated ship structure multiobjective optimization problem. In the result of calculations, an approximation of the Pareto-optimal set containing in each simulation from a few to more than 10 nondominated solutions was found. The obtained results do not allow for unequivocal decision on the advantage of either of the examined 1) substitute scalar objective function aggregation strategies; 2) domination attributes including into selection process as well as 3) distance to asymptotic solution including into a selection process was best.

In the case of studying the influence of optimization objectives aggregation strategy, visual assessment of the shape of the

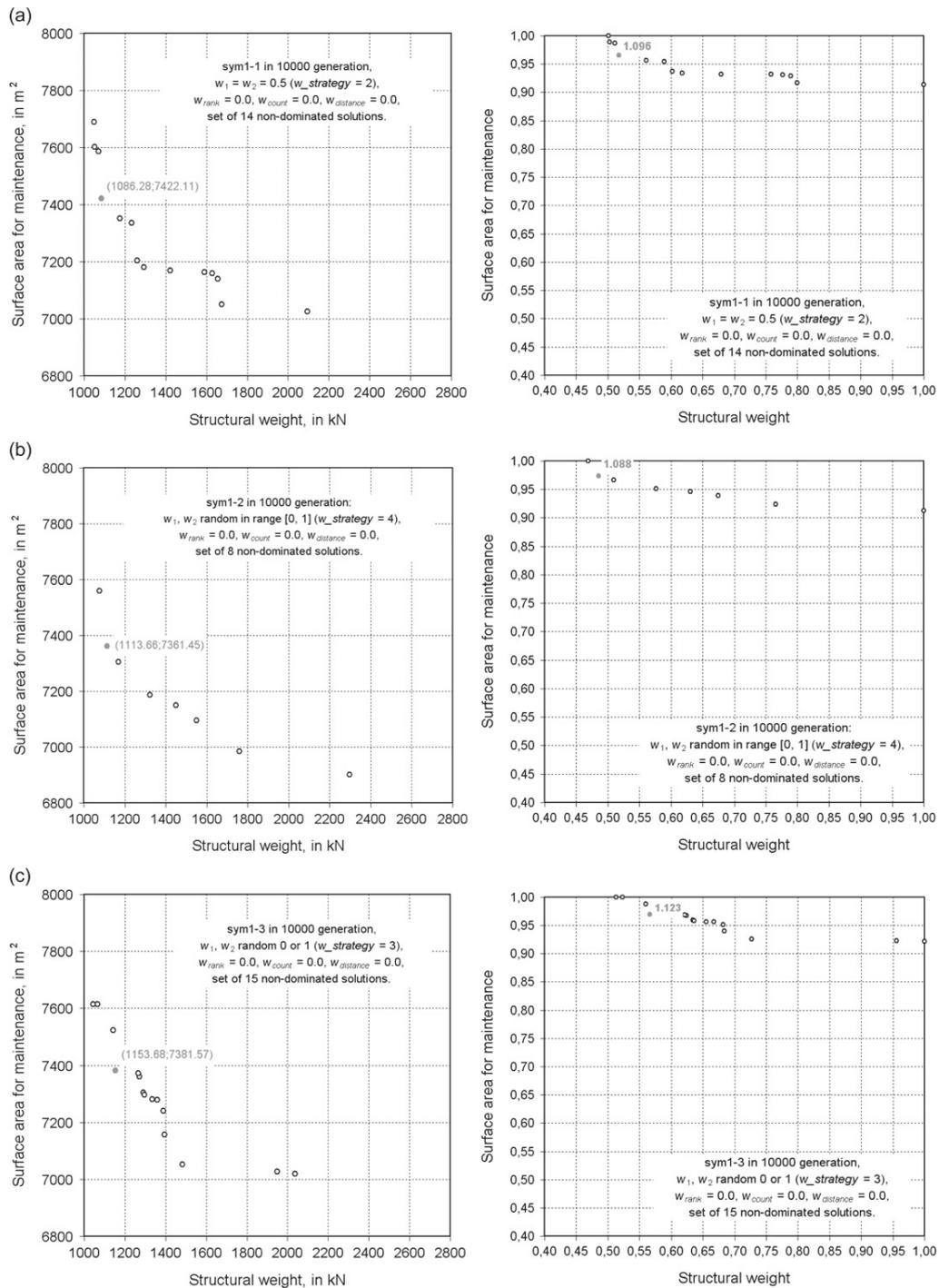


**Fig. 13** The results of multiobjective genetic optimization of ship structure with respect to structure weight  $f_1$  and the surface area of structural elements  $f_2$ : evolution of a highest value of fitness function  $f_{max}$  and the lowest value of nondominated solution distance from a asymptotic one  $\tilde{f}^*$ ; the values are dimensionless and standardized in  $[0,1]$  range in relation to the highest values found during the simulation; (a) sym1-1: in case of fixed values of optimization criteria weight coefficients  $w_1 = w_2 = 0.5$ ,  $w_{rank} = w_{count} = w_{distance} = 0.0$ ; (b) sym1-2: in case of random values of optimization criteria weight coefficients  $w_1$  and  $w_2$  in range of  $[0, 1]$ ,  $w_{rank} = w_{count} = w_{distance} = 0.0$ ; (c) sym1-3: in case of random values of weight coefficients  $w_1$  and  $w_2$  equaling 0 or 1,  $w_{rank} = w_{count} = w_{distance} = 0.0$ ; (d) sym2-1: in case of  $w_{rank} = 3.0$ ,  $w_{count} = w_{distance} = 0.0$ ,  $w_1 = w_2 = 0.0$ ; (e) sym2-2: in case of  $w_{count} = 3.0$ ,  $w_{rank} = w_{distance} = 0.0$ ,  $w_1 = w_2 = 0.0$ ; (f) sym2-3: in case of  $w_{distance} = 3.0$ ,  $w_{rank} = w_{count} = 0.0$ ,  $w_1 = w_2 = 0.0$

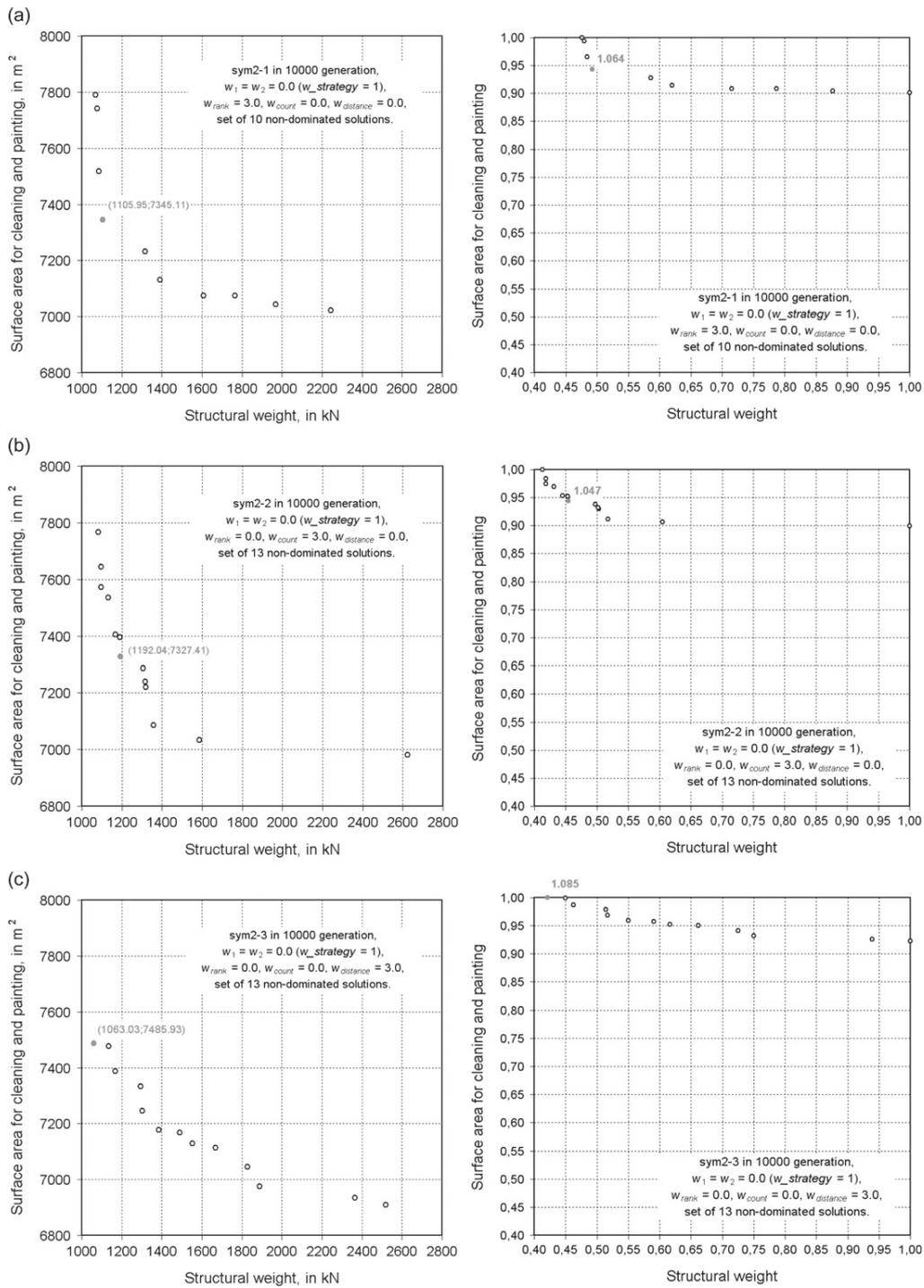
obtained of approximations of the Pareto-optimal set suggests an advantage of the strategy with random values of weight coefficients  $w_s$  (sym1-2) and the least effectiveness of the strategy with fixed values of the weight coefficients  $w_s$  (sym1-1). Effectiveness of the strategy with random selection of a single optimization objective in the individuals selection process (sym1-3) is intermediate. In the case of the constrained problems, it also turns that the components of the penalty func-

tions introduce a random contribution to the fitness function, thus causing the strategy with fixed weight coefficients  $w_s$  to be practically the strategy similar to the two others and it also allows to find the approximation of the Pareto-optimal set with the adequate accuracy.

From the sets of the compromise solutions, a user can in the next stage select one or a few solutions applying additional premises, which are not included in the optimization model. He or



**Fig. 14** Specification of the sets of nondominated solutions, in physical and normalized objective spaces, obtained during the performed genetic multiobjective optimization simulations of ship structure with respect to structure weight  $f_1$  and surface area  $f_2$ ; circles represent nondominated solutions, dots represent nondominated solutions closest to the asymptotic one; (a) sym1-1: in case of fixed values of optimization objectives, weight coefficients  $w_1 = w_2 = 0.5$ ,  $w_{rank} = w_{count} = w_{distance} = 0.0$ ; (b) sym1-2: in case of random values of optimization objectives weight coefficients  $w_1$  and  $w_2$  in range of  $[0, 1]$ ,  $w_{rank} = w_{count} = w_{distance} = 0.0$ ; (c) sym1-3: in case of random values of optimization objectives, weight coefficients  $w_1$  and  $w_2$  equaling 0 or 1,  $w_{rank} = w_{count} = w_{distance} = 0.0$



**Fig. 15** Specification of the sets of nondominated solutions, in physical and in normalized objective space, obtained during the performed genetic multiobjective optimization simulations of ship structure with respect to structure weight  $f_1$  and surface area  $f_2$ ; circles represent nondominated solutions, dots represent nondominated solutions closest to the asymptotic one; (a) sym2-1: in case of  $w_{rank} = 3.0$ ,  $w_{count} = w_{distance} = 0.0$ ,  $w_1 = w_2 = 0.0$ ; (b) sym2-2: in case of  $w_{count} = 3.0$ ,  $w_{rank} = w_{distance} = 0.0$ ,  $w_1 = w_2 = 0.0$ ; (c) sym2-3: in case of  $w_{distance} = 3.0$ ,  $w_{rank} = w_{count} = 0.0$ ,  $w_1 = w_2 = 0.0$

she can also select suggested nondominated solutions the closest to the asymptotic solution  $\mathbf{f}^{\approx}$ :

$$\mathbf{f}_{\text{sym1-1}}^{\approx} = [1086.28 \text{ kN } 7422.11 \text{ m}^2]^T \Rightarrow$$

$$f_{1,\text{sym1-1}}(\mathbf{x}) = 1086.28 \text{ kN}, f_{2,\text{sym1-1}}(\mathbf{x}) = 7422.11 \text{ m}^2,$$

$$\mathbf{f}_{\text{sym1-2}}^{\approx} = [1113.65 \text{ kN } 7361.45 \text{ m}^2]^T \Rightarrow$$

$$f_{1,\text{sym1-2}}(\mathbf{x}) = 1113.65 \text{ kN}, f_{2,\text{sym1-2}}(\mathbf{x}) = 7361.45 \text{ m}^2,$$

$$\mathbf{f}_{\text{sym1-3}}^{\approx} = [1153.68 \text{ kN } 7381.57 \text{ m}^2]^T \Rightarrow$$

$$f_{1,\text{sym1-3}}(\mathbf{x}) = 1153.68 \text{ kN}, f_{2,\text{sym1-3}}(\mathbf{x}) = 7381.57 \text{ m}^2.$$

The issue of examining the effectiveness of multiobjective evolutionary algorithm is very relevant, not solved all the way through and requires separate research, which exceeds content of this case study.

The analysis of the computer simulation results, for the formulated problem of the multiobjective optimization of ship structure, allows to formulate a conclusion that in the studied cases the most effective strategies were with random values of weight coefficients  $w_1$  and  $w_2$  in range of  $[0, 1]$ , and with fixed values of optimization objectives weight coefficients  $w_1 = w_2 = 0.5$ . Less effective was the strategy with random values of weight coefficients  $w_1$  and  $w_2$  equaling 0 or 1.

The recommended nondominated solution of the ship structure was specified in form of the ship cross-section given in Fig. 16.

For the study of dominance attributes influence and the distance from asymptotic solution on the effectiveness of the algorithm, sym2-1, sym2-2 and sym2-3, also satisfactory results were achieved:

$$\mathbf{f}_{\text{sym2-1}}^{\approx} = [1105.95 \text{ kN } 7345.11 \text{ m}^2]^T \Rightarrow$$

$$f_{1,\text{sym2-1}}(\mathbf{x}) = 1105.95 \text{ kN}, f_{2,\text{sym2-1}}(\mathbf{x}) = 7345.11 \text{ m}^2,$$

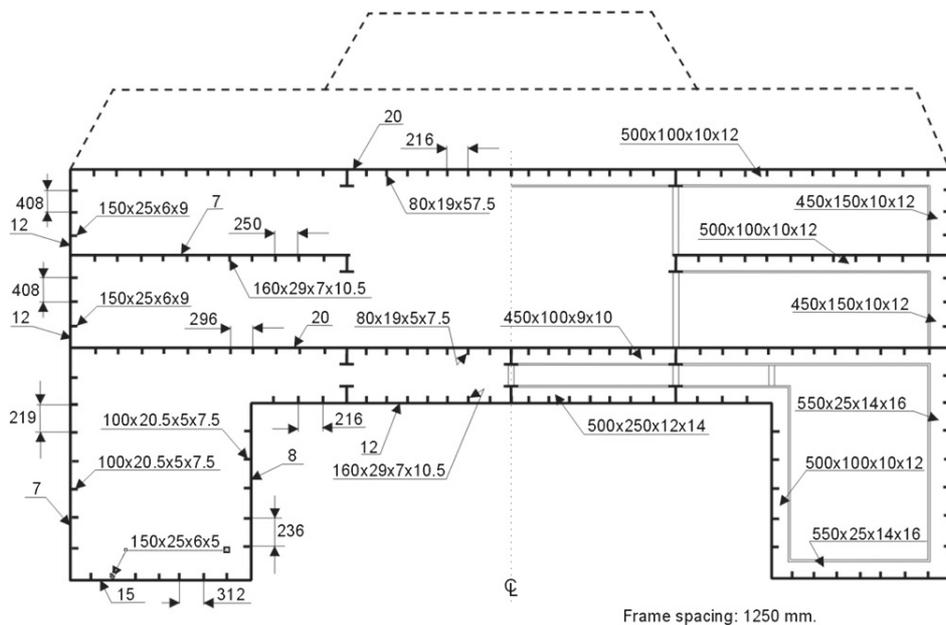
$$\mathbf{f}_{\text{sym2-2}}^{\approx} = [1192.04 \text{ kN } 7327.41 \text{ m}^2]^T \Rightarrow$$

$$f_{1,\text{sym2-2}}(\mathbf{x}) = 1192.04 \text{ kN}, f_{2,\text{sym2-2}}(\mathbf{x}) = 7327.41 \text{ m}^2,$$

$$\mathbf{f}_{\text{sym2-3}}^{\approx} = [1060.03 \text{ kN } 7485.93 \text{ m}^2]^T \Rightarrow$$

$$f_{1,\text{sym2-3}}(\mathbf{x}) = 1060.03 \text{ kN}, f_{2,\text{sym2-3}}(\mathbf{x}) = 7485.93 \text{ m}^2.$$

The computational investigations have positively verified the effectiveness of the combined fitness multiobjective evolutionary algorithm developed by the author as well as the computer code for solution of the unified topology-sized ship structure multiobjective optimization problem. The computer simulations have found approximately 12 nondominated solutions, which constitute the tradeoff solutions set, from which decision-makers may choose one or more of them for further development. The algorithm developed as a part of the underlying work allows also for pinpointing a single variant closest to the asymptotic solution, which may be proposed as the single solution of the multiobjective optimization problem.



**Fig. 16** Dimensions and scantlings of vessel structure recommended as result of multiobjective optimization; structural material: 5083-H111 aluminium alloys for plates, 6082-T6 aluminium alloys for profile extrusions (all dimensions are in millimeters)

## 8. Summary and conclusions

The problem of weight and total surface area minimization for a three-dimensional full midship block-section of the high speed catamaran hull was presented and discussed in detail. Strength criteria for ship structure checking were taken from the adopted classification rules. A computer code for solution of the formulated unified problem of the multiobjective optimization of topology and scantlings of the seagoing ship hull dimensions with the accuracy typical for the initial design was developed.

The application of the genetic algorithm concept to solve the formulated optimization problem was presented. It was proven in the study that the GA allows to include in the multiobjective optimization model a large number of design variables of the real ship structure. Introducing constraints related to strength, fabrication, and standardization is not difficult and could cover a more representative set of the objectives.

The aggregation method was proved effective even in the case of the fixed values of the weight coefficients because in the case of the constrained problem, the components of the penalty function introduce a random influence to the fitness function. The method is thus closer to the method using the random weight coefficients of the optimization objectives.

The author discussed the crucial role of the Pareto domination relation in process of evolution of ship structure feasible solutions toward the Pareto front containing the nondominated variants of the ship structure. Using the concept of domination in feasible solutions set, the definitions were proposed for the concept of the domination rank as well as domination count to take into account the relation of feasible variant with regard to other feasible variants. Using these ideas, the author suggested a version of the evolutionary algorithm for solution of the multiobjective, topology-sized optimization problem of a seagoing ship hull structure. In the selection process, the combined fitness function was applied allowing for taking into account in selection process: 1) optimization objectives; 2) dominance attributes; 3) distance to the asymptotic solution as well as 4) penalty functions for violating adopted constraints.

Computer code was developed for ship structural multiobjective optimization with the accuracy appropriate for the preliminary design. Using the code and hull structural model of a fast passenger-vehicle ferry built in the catamaran configuration according to Auto Express 82 design, series of the computer simulations were done.

The results of the computations allow to formulate the conclusion that the present algorithm can be an effective tool for the multiobjective optimization of ship structures in the early design stage. Application of the algorithm in the ship structural design can support the designer in proper identification of the set of "best possible compromises" or the single "best possible solution" as a result of the multiobjective ship structure design process. The set of "best possible compromises" means here the approximation of the Pareto set produced by the present algorithm. However, as the "best possible solution," the solution closest to the asymptotic solution can be taken.

The developed multiobjective optimization algorithm is based on the random processes; therefore, the obtained results should be interpreted in the statistical sense. It means that the simulations and obtained results may be not representative. Small change of the developed models or control parameters may result in a differ-

ent simulation course and different results. Therefore, further systematic studies of the algorithm efficiency controlled by particular components of combined fitness function are necessary.

Further systematic investigations of effectiveness of the proposed strategies including repeated computations varying only in the evolution history for statistic confirmation of effectiveness of the strategies are required.

The important part of the following investigation will be to include the procedures for verification of structural strength using more advanced methods than the applied approach of approximate computations using the equations from the classification society rules.

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