Synthesis of the Elements in Stars*

E. MARGARET BURBIDGE, G. R. BURBIDGE, WILLIAM A. FOWLER, AND F. HOYLE

Kellogg Radiation Laboratory, California Institute of Technology, and
Mount Wilson and Palomar Observatories, Carnegie Institution of Washington,
California Institute of Technology, Pasadena, California

"It is the stars, The stars above us, govern our conditions”;
(King Lear, Act IV, Scene 3)

but perhaps

"The fault, dear Brutus, is not in our stars, But in ourselves,”
(Julius Caesar, Act I, Scene 2)

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I. INTRODUCTION

A. Element Abundances and Nuclear Structure

MAN inhabits a universe composed of a great variety of elements and their isotopes. In Table I, a count of the stable and radioactive elements and isotopes is listed. Ninety elements are found terrestrially and one more, technetium, is found in stars; only promethium has not been found in nature. Some 272 stable and 55 naturally radioactive isotopes occur on the earth. In addition, man has been able to produce artificially the neutron, technetium, promethium, and ten transuranic elements. The number of radioactive isotopes he has produced now numbers 871 and this number is gradually increasing.

Each isotopic form of an element contains a nucleus with its own characteristic nuclear properties which are different from those of all other nuclei. Thus the total number of known nuclear species is almost 1200, with some 327 of this number known to occur in nature. In spite of this, the situation is not as complex as it might seem. Research in "classical" nuclear physics since 1932 has shown that all nuclei consist of two fundamental building blocks. These are the proton and the neutron which are called nucleons in this context. As long as energies below the meson production threshold are not exceeded, all "prompt" nuclear processes can be described as the shuffling and reshuffling of protons and neutrons into the variety of nucleonic packs called nuclei. Only in the slow beta-decay processes is there any interchange between protons and neutrons at low energies, and even there, as in the prompt reactions, the number of nucleons remains constant. Only at very high energies can nucleons be produced or annihilated. Prompt nuclear processes plus the slow beta reactions make it possible in principle to transmute any one type of
nuclear material into any other even at low energies of interaction.

With this relatively simple picture of the structure and interactions of the nuclei of the elements in mind, it is natural to attempt to explain their origin by a synthesis or buildup starting with one or the other or both of the fundamental building blocks. The following question can be asked: What has been the history of the matter, on which we can make observations, which produced the elements and isotopes of that matter in the abundance distribution which observation yields? This history is hidden in the abundance distribution of the elements. To attempt to understand the sequence of events leading to the formation of the elements it is necessary to study the so-called universal or cosmic abundance curve.

Whether or not this abundance curve is universal is not the point here under discussion. It is the distribution for the matter on which we have been able to make observations. We can ask for the history of that particular matter. We can also seek the history of the peculiar and abnormal abundances, observed in some stars. We can finally approach the problem of the universal or cosmic abundances. To avoid any implication that the abundance curve is universal, when such an implication is irrelevant, we commonly refer to the number distribution of the atomic species as a function of atomic weight simply as the atomic abundance distribution. In graphical form, we call it the atomic abundance curve.

The first attempt to construct such an abundance curve was made by Goldschmidt (Go37). An improved curve was given by Brown (Br49) and more recently Suess and Urey (Su56) have used the latest available data to give the most comprehensive curve so far available. These curves are derived mainly from terrestrial, meteoritic, and solar data, and in some cases from other astronomical sources. Abundance determinations for

\[ A/\text{amu} = \text{Stable} \times 10^{22} \]  

the sun were first derived by Russell (Ru29) and the most recent work is due to Goldberg, Aller, and Müller (Go57). Accurate relative isotopic abundances are available from mass spectroscopic data, and powerful use was made of these by Suess and Urey in compiling their abundance table. This table, together with some solar values given by Goldberg et al., forms the basic data for this paper.

It seems probable that the elements all evolved from hydrogen, since the proton is stable while the neutron is not. Moreover, hydrogen is the most abundant element, and helium, which is the immediate product of hydrogen burning by the \( p p \) chain and the CN cycle, is the next most abundant element. The packing-fraction curve shows that the greatest stability is reached at iron and nickel. However, it seems probable that iron and nickel comprise less than 1% of the total mass of the galaxy. It is clear that although nuclei are tending to evolve to the configurations of greatest stability, they are still a long way from reaching this situation.

It has been generally stated that the atomic abundance curve has an exponential decline to \( A \sim 100 \) and is approximately constant thereafter. Although this is very roughly true it ignores many details which are important clues to our understanding of element synthesis. These details are shown schematically in Fig. I,1

### Table I,1. Table of elements and isotopes [compiled from Chart of the Nuclides (Knolls Atomic Power Laboratory, April, 1956)].

<table>
<thead>
<tr>
<th>Elements</th>
<th>Isotopes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stable</td>
<td>81 Stable</td>
</tr>
<tr>
<td>Radioactive:</td>
<td>Radioactive:</td>
</tr>
<tr>
<td>Natural ((Z \leq 83))</td>
<td>1 (_{a}) Natural ((A &lt; 206))</td>
</tr>
<tr>
<td>((Z &gt; 83))</td>
<td>9 (_{b}) ((A \geq 206))</td>
</tr>
<tr>
<td>Natural:</td>
<td>Stable and Radioactive</td>
</tr>
<tr>
<td>Radioactive:</td>
<td>Artificial ((Z \leq 83))</td>
</tr>
<tr>
<td>((Z &gt; 83))</td>
<td>10 (_{a}) ((A \geq 206))</td>
</tr>
<tr>
<td>Total</td>
<td>102 Total</td>
</tr>
<tr>
<td>Neutron</td>
<td>1 Neutron</td>
</tr>
<tr>
<td>103</td>
<td>1199</td>
</tr>
</tbody>
</table>

\(_{a}\) Tc, observed in S-type stars.

\(_{b}\) Including At and Fr produced in weak side links of natural radioactivity.

\(_{c}\) Neutron, not observed in nature.

\(_{d}\) Including Hg, C, and Tc.

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\[ ^{\text{[1]}} \text{Refer to Bibliography at end of paper.} \]
Table I,2. Features of the abundance curve.

<table>
<thead>
<tr>
<th>Feature</th>
<th>Cause</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exponential decrease from hydrogen to $A \sim 100$</td>
<td>Increasing rarity of synthesis for increasing $A$, reflecting that stellar evolution to advanced stages necessary to build high $A$ is not common.</td>
</tr>
<tr>
<td>Fairly abrupt change to small slope for $A &gt; 100$</td>
<td>Constant $\sigma(n,\gamma)$ in $s$ process. Cycling in $r$ process.</td>
</tr>
<tr>
<td>Rarity of D, Li, Be, B as compared with their neighbors H, He, C, N, O</td>
<td>Inefficient production, also consumed in stellar interiors even at relatively low temperatures.</td>
</tr>
<tr>
<td>High abundances of alpha-particle nuclei such as O$^{16}$, Ne$^{20}$...Ca$^{40}$, Ti$^{44}$ relative to their neighbors</td>
<td>He burning and $\alpha$ process more productive than $H$ burning and $s$ process in this region.</td>
</tr>
<tr>
<td>Strongly-marked peak in abundance curve centered on Fe$^{56}$</td>
<td>$\varepsilon$ process; stellar evolution to advanced stages where maximum energy is released ($\varepsilon$ process lies near minimum of packing-fraction curve).</td>
</tr>
<tr>
<td>Double peaks ${A = 80, 130, 196$ Neutron capture in $r$ process (magic $N = 50, 82, 126$ for progenitors). $A = 90, 138, 208$ Neutron capture in $s$ process (magic $N = 50, 82, 126$ for stable nuclei).</td>
<td></td>
</tr>
<tr>
<td>Rarity of proton-rich heavy nuclides</td>
<td>Not produced in main line of $r$ or $s$ process; produced in rare $p$ process.</td>
</tr>
</tbody>
</table>

and are outlined in the left-hand column of Table I,2. The explanation of the right-hand column will be given in Sec. II.

It is also necessary to provide an explanation of the origin of the naturally radioactive elements. Further, the existence of the shielded isobars presents a special problem.

B. Four Theories of the Origin of the Elements

Any completely satisfactory theory of element formation must explain in quantitative detail all of the features of the atomic abundance curve. Of the theories so far developed, three assume that the elements were built in a primordial state of the universe. These are the nonequilibrium theory of Gamow, Alpher, and Herman [see (Al50)], together with the recent modifications by Hayashi and Nishida (Ha56), the poly-neutron theory of Mayer and Teller (Ma49), and the equilibrium theory developed by Klein, Beskow, and Treffenberg (K147). A detailed review of the history and development of these theories was given by Alpher and Herman (Al53).

Each of these theories possesses some attractive features, but none succeeds in meeting all of the requirements. It is our view that these are mainly satisfied by the fourth theory in which it is proposed that the stars are the seat of origin of the elements. In contrast with the other theories which demand matter in a particular primordial state for which we have no evidence, this latter theory is intimately related to the known fact that nuclear transformations are currently taking place inside stars. This is a strong argument, since the primordial theories depend on very special initial conditions for the universe. Another general argument in favor of the stellar theory is as follows.

It is required that the elements, however they were formed, are distributed on a cosmic scale. Stars do this by ejecting material, the most efficient mechanisms being probably the explosive ejection of material in supernovae, the less energetic but more frequent novae, and the less rapid and less violent ejection from stars in the giant stages of evolution and from planetary nebulae. Primordial theories certainly distribute material on a cosmic scale but a difficulty is that the distribution is thought to have been spatially uniform and independent of time once the initial phases of the universe were past. This disagrees with observation. There are certainly differences in composition between stars of different ages, and also stars at particular evolutionary stages have abnormalities such as the presence of technetium in the $S$-type stars and $Cf^{254}$ in supernovae. A detailed discussion of these and other features is given in Secs. XI and XII.

It is not known for certain at the present time whether all of the atomic species heavier than hydrogen have been produced in stars without the necessity of element synthesis in a primordial explosive stage of the universe. Without attempting to give a definite answer to this problem we intend in this paper to restrict ourselves to element synthesis in stars and to lay the groundwork for future experimental, observational, and theoretical work which may ultimately provide conclusive evidence for the origin of the elements in stars. However, from the standpoint of the nuclear physics alone it is clear that our conclusions will be equally valid for a primordial synthesis in which the initial and later evolving conditions of temperature and density are similar to those found in the interiors of stars.

C. General Features of Stellar Synthesis

Except at catastrophic phases a star possesses a self-governing mechanism in which the temperature is adjusted so that the outflow of energy through the star is balanced by nuclear energy generation. The temperature required to give this adjustment depends on the particular nuclear fuel available. Hydrogen requires a lower temperature than helium; helium requires a lower temperature than carbon, and so on, the increasing temperature sequence ending at iron since energy generation by fusion processes ends here. If hydrogen is present the temperature is adjusted to hydrogen as a fuel, and is comparatively low. But if hydrogen becomes exhausted as stellar evolution proceeds, the temperature rises until helium becomes effective as a fuel. When helium becomes exhausted the temperature rises still
further until the next nuclear fuel comes into operation, and so on. The automatic temperature rise is brought about in each case by the conversion of gravitational energy into thermal energy.

In this way, one set of reactions after another is brought into operation, the sequence always being accompanied by rising temperature. Since penetrations of Coulomb barriers occur more readily as the temperature rises it can be anticipated that the sequence will be one in which reactions take place between nuclei with greater and greater nuclear charges. As it becomes possible to penetrate larger and larger barriers the nuclei will evolve towards configurations of greater and greater stability, so that heavier and heavier nuclei will be synthesized until iron is reached. Thus there must be a progressive conversion of light nuclei into heavier ones as the temperature rises.

There are a number of complicating factors which are superposed on these general trends. These include the following.

The details of the rising temperature and the barrier effects of nuclear reactions at low temperatures must be considered.

The temperature is not everywhere the same inside a star, so that the nuclear evolution is most advanced in the central regions and least or not at all advanced near the surface. Thus the composition of the star cannot be expected to be uniform throughout. A stellar explosion does not accordingly lead to the ejection of material of one definite composition, but instead a whole range of compositions may be expected.

Mixing within a star, whereby the central material is mixed outward, or the outer material inward, produces special effects.

Material ejected from one star may subsequently become condensed in another star. This again produces special nuclear effects.

All of these complications show that the stellar theory cannot be simple, and this may be a point in favor of the theory, since the abundance curve which we are trying to explain is also not simple. Our view is that the elements have evolved, and are evolving, by a whole series of processes. These are marked in the schematic abundance curve, Fig. I,1, as H burning, He burning, $\alpha$, $\epsilon$, $r$, $s$, and $p$ processes. The nature of these processes is shown in detail in Fig. I,2; details of this diagram are explained in the following sections.

II. PHYSICAL PROCESSES INVOLVED IN STELLAR SYNTHESIS, THEIR PLACE OF OCCURRENCE, AND THE TIME-SCALES ASSOCIATED WITH THEM

A. Modes of Element Synthesis

As was previously described in an introductory paper on this subject by Hoyle, Fowler, Burbidge, and Burbidge (Ho56), it appears that in order to explain all of the features of the abundance curve, at least eight different types of synthesizing processes are demanded, if we believe that only hydrogen is primeval. In order to clarify the later discussion we give an outline of these processes here (see also Ho54, Fo56).

(i) Hydrogen Burning

Hydrogen burning is responsible for the majority of the energy production in the stars. By hydrogen burning in element synthesis we shall mean the cycles which synthesize helium from hydrogen and which synthesize the isotopes of carbon, nitrogen, oxygen, fluorine, neon, and sodium which are not produced by processes (ii) and (iii). A detailed discussion of hydrogen burning is given in Sec. III.

(ii) Helium Burning

These processes are responsible for the synthesis of carbon from helium, and by further $\alpha$-particle addition for the production of $O^{16}$, $Ne^{20}$, and perhaps $Mg^{24}$. They are described in detail in Sec. III.

(iii) $\alpha$ Process

These processes include the reactions in which $\alpha$ particles are successively added to $Ne^{20}$ to synthesize the four-structure nuclei $Mg^{24}$, $Si^{28}$, $S^{32}$, $A^{38}$, $C^{40}$, and probably $Ca^{44}$ and $Ti^{46}$. This is also discussed in Sec. III. The source of the $\alpha$ particles is different in the $\alpha$ process than in helium burning.

(iv) $e$ Process

This is the so-called equilibrium process previously discussed by Hoyle (Ho46, Ho54) in which under conditions of very high temperature and density the elements comprising the iron peak in the abundance curve (vanadium, chromium, manganese, iron, cobalt, and nickel) are synthesized. This is considered in detail in Sec. IV.

(v) $s$ Process

This is the process of neutron capture with the emission of gamma radiation $(n,\gamma)$ which takes place on a long time-scale, ranging from $\sim 100$ years to $\sim 10^4$ years for each neutron capture. The neutron captures occur at a slow ($s$) rate compared to the intervening beta decays. This mode of synthesis is responsible for the production of the majority of the isotopes in the range $23 \leq A \leq 46$ (excluding those synthesized predominantly by the $\alpha$ process), and for a considerable proportion of the isotopes in the range $63 \leq A \leq 209$. Estimates of the time-scales in different regions of the neutron-capture chain in the $s$ process will be considered later in this section, while the details of the nuclear physics of the process are discussed in Secs. V and VI together with the results. The $s$ process produces the abundance peaks at $A = 90$, 138, and 208.
Fig. 1.2. A schematic diagram of the nuclear processes by which the synthesis of the elements in stars takes place. Elements synthesized by interactions with protons (hydrogen burning) are listed horizontally. Elements synthesized by interactions with alpha particles (helium burning) and by still more complicated processes are listed vertically. The details of the production of all of the known stable isotopes of carbon, nitrogen, oxygen, fluorine, neon, and sodium are shown completely. Neutron capture processes by which the highly charged heavy elements are synthesized are indicated by curved arrows. The production of radioactive Tc99 is indicated as an example for which there is astrophysical evidence of neutron captures at a slow rate over long periods of time in red giant stars. Similarly Cf254, produced in supernovae, is an example of neutron synthesis at a rapid rate. The iron group is produced by a variety of nuclear reactions at equilibrium in the last stable stage of a star's evolution.
SYNTHESIS OF ELEMENTS IN STARS

(iii) \( r \) Process

This is the process of neutron capture on a very short time-scale, \( \sim 0.01 \text{--} 10 \text{ sec} \) for the beta-decay processes interspersed between the neutron captures. The neutron captures occur at a rapid \( (r) \) rate compared to the beta decays. This mode of synthesis is responsible for production of a large number of isotopes in the range \( 70 \leq A \leq 209 \), and also for synthesis of uranium and thorium. This process may also be responsible for some light element synthesis, e.g., \( \text{Na}^{22} \), \( \text{Ca}^{44} \), \( \text{Ca}^{40} \), and perhaps \( \text{Ti}^{47} \), \( \text{Ti}^{49} \), and \( \text{Ti}^{50} \). Details of this process and the results of the calculations are discussed in Secs. VII and VIII. The \( r \) process produces the abundance peaks at \( A = 80 \), 130, and 194.

(vii) \( p \) Process

This is the process of proton capture with the emission of gamma radiation \( \gamma \) or the emission of a neutron following gamma-ray absorption \( \gamma \), which is responsible for the synthesis of a number of proton-rich isotopes having low abundances as compared with the nearby normal and neutron-rich isotopes. It is discussed in Sec. IX.

(viii) \( x \) Process

This process is responsible for the synthesis of deuterium, lithium, beryllium, and boron. More than one type of process may be demanded here (described collectively as the \( x \) process), but the characteristic of all of these elements is that they are very unstable at the temperatures of stellar interiors, so that it appears probable that they have been produced in regions of low density and temperature. There is, however, some observational evidence against this which is discussed in Sec. X together with the details of the possible synthesizing processes.

In the upper half of Table II,1 the abundances of different natural groups of elements, taken from the atomic abundance table of Suess and Urey (Su56), have been summed and listed, firstly by number and by fraction of the total, and secondly by weight and by mass fraction of the total. In the lower half of Table II,1, a similar listing has been made, but in this case the isotopes have been grouped according to which mode of synthesis has been responsible for their production. Our method of assignment to these different processes is described in subsection B. In some cases a natural group comprises just the elements built by one process; e.g., the iron group of elements are also the \( e \)-process isotopes in the lower half of the table, and consequently the same entry appears in both halves. In one case, that of the \( r \)-process isotopes of intermediate atomic weight, the estimate given is one that has been calculated in Sec. VII.

An auxiliary but indispensable process which is also demanded in our description of element synthesis is a nuclear process which will provide a source of free neutrons for both the \( s \) process and the \( r \) process. The first suggestion in this direction was made by Cameron (Ca54, Ca55) and Greenstein (Gr54), who proposed that the \( \text{C}^4 \alpha (\alpha, n) \text{O} \) reaction would provide such a source. In addition to this the reaction \( \text{Ne}^{23}(\alpha, n) \text{Mg}^{24} \) has also been proposed by Fowler, Burbidge, and Burbidge (Fo55). More detailed work on the rates of these reactions by Marion and Fowler is now available (Ma57) and some discussion is included in Sec. III.

B. Method of Assignment of Isotopes among Processes (i) to (viii)

Of the eight processes which are demanded to synthesize all of the stable isotopes, assignments among hydrogen burning, helium burning, the \( \alpha \) process, the \( e \) process, and the \( x \) process are comparatively straightforward, and are implicit in previous work and in the discussions in Secs. III, IV, and X. Thus pure hydrogen

| Table II,1. Atomic abundances of various groups of the elements from Suess and Urey (Su56). |
|-------------------|-------------------|-------------------|-------------------|
| Group             | By number         | Fraction of total | By weight         | Fraction of total |
| H                 | \( 4.00 \times 10^3 \) | 0.928             | \( 4.03 \times 10^6 \) | 0.755             |
| He                | \( 3.08 \times 10^6 \) | 0.071             | \( 1.23 \times 10^9 \) | 0.231             |
| Li, Be, B         | 1.44 \times 10^9   | 3.3 \times 10^{-9} | 1.30 \times 10^9   | 2.4 \times 10^{-4} |
| Carbon group: C, N, O, Ne | 4.01 \times 10^7 | 9.5 \times 10^{-4} | 6.5 \times 10^9   | 1.2 \times 10^{-8} |
| Silicon group: Na–Sc | 2.65 \times 10^6 | 6.1 \times 10^{-5} | 7.3 \times 10^9   | 1.3 \times 10^{-7} |
| Iron group: 50 ≤ A ≤ 62 | 6.4 \times 10^8 | 1.5 \times 10^{-4} | 3.6 \times 10^9 | 6.7 \times 10^{-4} |
| Middleweight: 63 ≤ A ≤ 100 | 1.1 \times 10^8 | 2.6 \times 10^{-4} | 7.7 \times 10^9 | 1.4 \times 10^{-4} |
| Heavyweight: A ≥ 100 | 28               | 6.5 \times 10^{-10} | 4.6 \times 10^9 | 8.6 \times 10^{-4} |
| H+He burning: 12 ≤ A ≤ 22 | 4.01 \times 10^7 | 9.3 \times 10^{-4} | 6.5 \times 10^9 | 1.2 \times 10^{-4} |
| \( \alpha \) process: 24, 28, ... 48 | 2.2 \times 10^6 | 5.1 \times 10^{-4} | 6.1 \times 10^9 | 1.1 \times 10^{-4} |
| \( s \) process: 23 ≤ A ≤ 46 | 4.7 \times 10^6 | 1.1 \times 10^{-4} | 1.3 \times 10^9 | 2.4 \times 10^{-4} |
| \( e \) process: 50 ≤ A ≤ 62 | 6.4 \times 10^6 | 1.5 \times 10^{-4} | 3.6 \times 10^9 | 6.7 \times 10^{-4} |
| \( s \) process: 63 ≤ A ≤ 75 | 8.8 \times 10^9 | 2.0 \times 10^{-4} | 5.7 \times 10^9 | 1.1 \times 10^{-7} |
| \( r \) process: A ≥ 75 | 1.1 \times 10^8 | 2.6 \times 10^{-4} | 1.1 \times 10^9 | 2.1 \times 10^{-4} |
| \( p \) process: | 3.1           | 3.5 \times 10^{-4} | 1.4 \times 10^9 | 2.6 \times 10^{-7} |
| Summary: X(H) = 0.755, Y(He) = 0.231, Z(A > 4) = 0.014 | | | | |
burning is responsible for the synthesis of helium, and
pure helium burning is responsible for C\(^{22}\), O\(^{16}\),
and Ne\(^{20}\). Also, hydrogen burning in these latter products of helium burning accounts for all of the stable isotopes up to Ne\(^{22}\) (cf. Sec. III). The only exceptions up to this point are deuterium, lithium, beryllium, and boron, which have therefore been assigned to a radically different type of synthesizing process, the x process. Assignment to the \(\alpha\) process for a number of four-structure
nuclei was originally suggested by Hoyle (Ho54). All of
these nuclei lie in a region of the abundance curve in
which the surrounding nuclei are synthesized by the
\(\beta\) process. As described in detail in Sec. VI, a plot of the
\(\alpha\)N products (neutron capture cross section times
abundance) for such nuclei should delineate a smooth curve.
However, because of their comparatively large abundances,
the nuclei made by the \(\alpha\) process all lie above this
curve. This result bears out the assignment of these
nuclei to the \(\alpha\) process. Assignment of the nuclei in the
iron peak to the \(\beta\) process was also proposed earlier by
Hoyle. The large abundance peak at this point shows
clearly that a separate process is demanded and the
results of Sec. IV suggest strongly that our assignments
to the \(\beta\) process are correct.

As far as assignments among the \(\alpha\), \(\beta\), and \(\gamma\) processes
are concerned, the situation is a little more complex.
Suess and Urey (Su56) and Coryell (Cu56) have already
pointed out that the peaks in the abundance curves at
stable nuclei with filled neutron shells \((A=90, N=50;
A=138, N=82; A=208, N=126)\) strongly indicate
the operation of the \(\alpha\) process, and the nearby peaks
at \(A=80, 130, 194\), shifted by \(\delta A \approx 8\) to 14,
similarly require the operation of the \(\beta\) process. Also, calculations of Fowler et al. (Fo55) suggested that apart
from the nuclei built by the \(\alpha\) process all of the nuclei
with \(23 \leq A \leq 46\) with the exception of S\(^{32}\), Ca\(^{48}\),
and Ca\(^{48}\) can be synthesized by the \(\alpha\) process.

Certain isotopes of the heavy elements can be built
only by the \(\beta\) process while others can be built only by
the \(\alpha\) process. The two processes differ in this respect
because they allow very different times for the occurrence
of the beta disintegrations along the chain of
nuclei built by the neutron addition. In the \(\alpha\) process
most beta-active nuclei have time to decay before additional
capture occurs. On the other hand, the \(\beta\) process
involves neutron captures which take place at a rate
much faster than the beta decays which have reaction
times of 0.01 to 10 sec.

The need for a third process, the \(\gamma\) process, arises
because a third set of isotopes cannot be built by either
the rapid or the slow capture of neutrinos (the reasons
for this from the standpoint of nuclear structure are
discussed in Sec. V). As an example of the way in which
the three processes operate and also of how assignments
have been made, we shall consider the light isotopes of
tellurium. The light isotopes S\(_{55}\)Te\(_{54}\), S\(_{55}\)Te\(_{56}\),
and S\(_{55}\)Te\(_{58}\) can be synthesized only in the \(\alpha\) process, since
in the \(\beta\) process, the ultimate beta decays of the neutron-rich isotopes at 122, 123, 124, produced by rapid neutron
addition, terminate at the stable isotopes S\(_{56}\)Te\(_{52}\),
S\(_{56}\)Te\(_{54}\), and S\(_{56}\)Te\(_{56}\), which are on the neutron side
of the nuclear stability line. On the other hand, the
heaviest isotope Te\(_{58}\) can be produced only in the \(\beta\) process, where it is the stable product of the decay of
neutron-rich isotopes of mass 130. In the \(\beta\) process, radioactive Te\(_{58}\) with a half-life of 70 min has time to decay
to P\(_{50}\) (half-life 2\(\times\)10\(^{7}\) years), and after another neutron
capture, the resultant $^130$ I decays in 12.6 hr to $^{130}$ Xe, which is thus produced in the s chain instead of $^{130}$ Te. The isotopes $^{128}$ Te, $^{128}$ Te, and $^{128}$ Te can be produced in either the s or the r process, although $^{128}$ Te is produced in the slow capture of neutrons only in a weak side link of the chain resulting from the fact that $^{128}$ I decays 5% of the time by positron emission or electron capture. Thus we believe that it is synthesized predominantly by the r process. The rarest and lightest isotope $^{130}$ I cannot be built by either process and it is for this isotope that the s process is demanded. $^{130}$ Te is about 1% as abundant, and $^{132}$ Te, $^{132}$ Te, and $^{132}$ Te are about 10% as abundant, as $^{138}$ Te and $^{138}$ I. This suggests that we assign $^{138}$ Te, which has an abundance comparable to $^{138}$ Te and $^{138}$ I, to the r process. $^{138}$ Te is an intermediate case, but it follows the trend of the r process and to this we assign its production. Assignments between the s, r, and p processes have been made in this way. In Figs. II,1; II,2; and II,3, the separation of the isotopes in the region $120 < A < 150$ is shown. In Fig. II,1 the abundances are plotted logarithmically after the manner of Suess and Urey (Su56). Nuclear species produced in the same process are connected by shaded curves and the general trend of the production becomes clear. In order to show more clearly the great increase in abundance in the peaks, linear plots of the abundances of the odd and even isotopes, respectively, are shown in Figs. II,2 and II,3. The magic-number peaks stand out clearly in both odd-A and even-A nuclei.

C. Abundances and Synthesis Assignments

GIVEN IN THE APPENDIX

The Appendix contains all of the information we have been able to collect which is relevant to the synthesis problem. All of the stable isotopes in order of increasing A are given in this table. We also include the beta-unstable isotopes which lie on the main neutron-capture chain in the s process, together with a few others which are given for special reasons. The table was originally drawn up to represent the building of elements by the s process. For this reason the left-hand columns follow the main chain of nuclei synthesized by neutron capture in the s process. The right-hand columns give information concerning the isotopes which are either completely by-passed by the s process or which lie in the subsidiary loops of the chain. These loops can be formed in two ways. Either they form at nuclei which beta decay with a half-life such that in the s process a proportion of the nuclei will capture a further neutron before beta decay, while the remainder beta decay directly (the next step in the chain following the nucleus $X_{A-1}$ being either $X_{A-1}X_{A+1}$ or $X_{A-1}X_{A+1}$). Alternatively a loop will be formed if a nucleus can decay either by emitting an electron or by emitting a positron or capturing a K electron (the next step in the chain following the nucleus $X_{A-1}$ being either $X_{A-1}X_{A+1}$ or $X_{A-1}X_{A+1}$). A few isomeric states are also given [an example is $^{48}$ Cd(s)] in cases in which they also lead to the development of loops. In all cases the side of the loop which is in the main chain will be determined by the faster of the reactions (either neutron-capture or beta-decay) which can take place. To distinguish between nuclei which lie in the weak branch of a loop, and those which are completely by-passed.
in the chain, horizontal lines are drawn to contain each loop.

Although the table was prepared primarily for the $s$ process it is now used to describe all of the modes of element synthesis. For this reason a number of beta-active nuclei which are by-passed in regions of small $A$ are given. This enables us to discuss the $r$ process as it would apply for the very light elements.

Abundances given in the Appendix have been taken predominantly from the table of Suess and Urey (Su56). For the elements scandium, titanium, vanadium, chromium, manganese, iron, cobalt, nickel, copper, and lead, the solar abundances obtained by Goldberg, Aller, and Müller (Go57) are also given in parentheses under those of Suess and Urey. The product of the neutron-capture cross section times the solar abundance is also given in parentheses in the $eN$ column (cf. Sec. VI).

Assignments of all of the stable isotopes to their most probable mode of production are given in the right-hand columns in the table. In the $s$ and the $r$ processes, when the isotope can be made only by one or other of these processes, then they are designated $s$ only or $r$ only. In cases where the isotope is made predominantly by one process though the other may contribute a little—as, for example, where the $r$ process is mainly responsible for synthesis, though the isotope lies in the main $s$ chain—then we write simply $r$ (or $s$), hydrogen burning, helium burning, $\alpha$, $e$, $p$, $x$, as the case may be. In some cases more than one process may contribute appreciably to the synthesis of that isotope. In these cases the process which is believed to contribute most is written first, while in the few cases in which the two processes are believed to make equal contributions, an approximate equality sign is used, and in the abundance column ($\ddagger$) is written. In some other cases $\ddagger$ of the abundance of an isotope has been attributed to the $s$ process and only $\ddagger$ to the $r$ process. In these cases ($\ddagger$) is written in the abundance column (these fractions are only very approximate). Thus, in proceeding from isotopes which can be made solely on the $s$ process to those which are made solely on the $r$ process, the sequence of designations is $s$ only, $s$, $sr$, $s=r$, $rs$, $r$, $r$ only. Magic-number nuclei with closed shells having 14, 20, 28, 50, 82, and 126 neutrons, most of which are synthesized by the $s$ process, are marked in the assignment column of the table by $m$ in parentheses. Similarly, nuclei made by the $r$ process whose progenitors had closed shells having 50, 82, or 126 neutrons are marked by $m'$ in parentheses. A few of the light nuclei undergo nuclear reactions which lead not to heavier element synthesis but to breakup, or to decrease in $A$ (e.g., Li$^7$). In the table these have been designated return (meaning that they return back down the chain), and in some cases the products of the reactions which are responsible for this are given.

D. Time-Scales for Different Modes of Synthesis

(1) Hydrogen-Burning Time-Scale

Hydrogen burning is responsible for all of the energy production of stars on the main sequence. After stars have evolved off the main sequence following the development of a chemical inhomogeneity, hydrogen burning in a shell still remains an important energy source. Thus the synthesis of elements in hydrogen burning is going on continuously, and the range in timescales for particular stars is dependent only on their initial masses after condensation, and the point in their evolution at which they eject material which has been synthesized. Thus these time-scales may range from $\sim 10^9$ years for massive O and B stars to times which are of the order of though less than the age of the galaxy (since we have no evidence for the existence today of primeval stars of pure hydrogen).

(2) Helium-Burning Time-Scale

It is believed that helium burning takes place in stars which have evolved onto the giant branch of the $HR$ diagram. In this region central temperatures of $\sim 10^8$ degrees and densities of $\sim 10^5$ g/cc are reached in the helium core, according to the theoretical calculations of Hoyle and Schwarzschild (Ho55), and under these conditions synthesis of C$^\alpha$ becomes possible (Sa52, Co57, Sa57). The further helium-burning reactions leading to production of O$^{16}$, Ne$^{20}$, and Mg$^{24}$ also can take place under suitable conditions though the amounts of these isotopes synthesized will decrease as the Coulomb barriers get larger (cf. Sec. III). The timescale for helium burning is therefore governed by the lifetime of stars subsequent to their becoming red giants. Since their mode of evolution off the giant branch is not yet understood, it is difficult to make accurate estimates of this time, but calculations of Hoyle and Schwarzschild and Hoyle and Haselgrove (Ho56a) suggest that a time-scale of $\sim 10^7$-$10^8$ years for stars to evolve in this region is reasonable.

(3) $\alpha$-Process Time-Scale

This process, since it involves reactions between nuclei of comparatively large $Z$, demands temperatures of $\sim 10^9$ degrees (cf. Sec. III). Thus it appears that this condition will be reached only when a star is contracting further following the helium burning mentioned above. Consequently we may hypothesize that this situation is reached at a later stage of evolution, after the star has left the giant branch. The timescale involved is difficult to estimate. It is probably short in comparison with the time taken for the star to evolve in the giant region but long compared with the time-scale for the $e$, $r$, and $p$ processes. An argument concerning the synthesis of Ti$^{46}$, the last nucleus which may be built by this process, suggests that the time-scale for its production from the preceding nucleus is $\gtrsim 20$ years. Details
of this argument are given in Sec. III D. The total time-scale for the whole $\alpha$ process might lie in the range $\sim 10^4$–$10^6$ years.

(4) Rapid Time-Scales

$\epsilon$ process.—As is discussed in Secs. IV and XII, this process takes place under extreme conditions of temperature and density, probably just prior to a supernova outburst. The time-scale involved may be of the order of seconds or minutes.

$r$ process.—The nuclear physics of this process demands that neutrons be added extremely rapidly, so that the total time-scale for the addition of a maximum of about 200 neutrons per iron nucleus is $\sim 10$–$100$ sec. It has previously been suggested that the spontaneous fission of $^{238}$Cf is responsible for the form of the decay light curves of supernovae of Type I which have an exponential form with a half-life near 55 days, so that this is strong evidence for the fact that the $r$ process takes place in such outbursts (Bu56). The time-scale for the explosive phases of such outbursts may well be as short as 100–1000 sec. Further support is given by a comparison of the results of calculations based on this hypothesis with the observed abundances of isotopes built by this process (Secs. VII and VIII).

$p$ process.—It has previously been suggested that this process takes place in the outbursts of supernovae of Type II on a time-scale comparable to that for the $r$ process.

(5) $s$-Process Time-Scale

In earlier work it was suggested that the $s$ process is currently going on in the interiors of some red giant stars (Ca55, Fo55). The presence of technetium in the atmospheres of $S$-type stars, observed by Merrill (Me52), and also to a lesser extent in carbon and $M$-type stars, affords conclusive evidence that nuclear activity involving neutron capture is currently going on in the interiors of these stars. This element is being both synthesized and mixed to the surface of the star in a time which is less than or at least of the same order as the half-life against beta decay of the longest-lived isotope of technetium ($^{99}$Tc with a half-life of $2\times10^3$ years). This is compatible with the result that the time taken for the stars to evolve in the giant branch of the HR diagram is $\sim 10^7$ years. Of the reactions $^{40}$Ca($n$,n)$^{40}$Ca and $^{22}$Ne($n$,n)$^{22}$Ne, which can give neutron production, the former will give an adequate supply of neutrons to carry heavy-element synthesis right through to the heaviest nuclei only if there is some mixing between the envelope hydrogen and the products of helium burning in the cores at the red-giant stage. In a previous paper (Fo55) it was pointed out that the $^{56}$Fe($n$,n)$^{56}$Fe reaction could provide an adequate source of neutrons and also avoid the problem of mixing between core and envelope. In Sec. III F, however, it is pointed out that for this to be the case, certain conditions must be satisfied. The relative importance of these two reactions for the $s$ process will be discussed in detail in Sec. III F. The evidence from work on theoretical models suggests that a very deep convective zone extending inward from the surface is not unreasonable, so that mixing of the synthesized material to the surface can take place.

As will be seen, there appear to be two time-scales for the $s$ process. These depend on the average times taken for the nuclei to capture neutrons. Thus these times are functions of the neutron density and hence of the rates of the neutron-producing reactions, and also of the capture cross sections. In Sec. V we describe the method of estimating the neutron capture cross sections ($\sigma$) given in the appendix. By using these together with the isotopic abundances and the assignments of nuclei to the $s$ process, it is possible to make some estimates of the neutron-capture times at particular points in the slow neutron-capture chain. The method of doing this is as follows.

As pointed out in Sec. II C, if the beta-decay time is greater than a few years then the next link in the chain following $X_{s}^4$ may be either $X_{s+1}^4$ or $X_{s+4}^0$, the relative importance of the branch depending on the time-scale for neutron capture at this point. We show in the appendix all possible loops corresponding to time-scales between $\sim 10^5$ years and $\sim 10^8$ years. Thus in all cases in which the beta decay takes place in a time $< 100$ years we have assumed that all of the nuclei will beta decay and any nuclei which can only be produced if another neutron is captured before the beta decay takes place are by-passed in the $s$ process. If on the other hand the half-life for beta decay is $\gg 10^8$ years we have assumed that the nucleus is effectively stable in the $s$ chain and will always capture a neutron. In cases like this it must be remembered, however, that all of the unstable nuclei with half-lives $\gg 10^8$ years which are effectively stable as far as the $s$ process is concerned will eventually decay into the succeeding stable nuclei long after the $s$ process has ceased operating. For elements in the solar system this will be true for all long-lived but unstable nuclei for which the half-life $< 5\times 10^8$ years.

There appear to be two indicators for the time-scale for the $s$ process in two regions of $A$. The first can be obtained from two isotopes of krypton. Both $^{80}$Kr and $^{88}$Kr are built only by the $s$ process, being shielded in the $r$ process by $^{80}$Se and $^{82}$Se. However, from the appendix we see that there is an $s$ chain loop at $^{83}$Se which is due to the beta decay of this isotope with a half-life of $7\times 10^8$ years. Thus while $^{80}$Kr is produced partly in the main chain and partly in the loop, $^{86}$Kr is only produced in the loop. Now under conditions of steady neutron flow (see Sec. VI) the basic equations for the relative abundances of these isotopes are

$$N(Kr^{80})\lambda_s(Kr^{80}) = 0.92N(Se^{79})\lambda_s(Se^{79})$$

and

$$N(Kr^{86})\lambda_s(Kr^{86}) = N(Se^{79})[\lambda_s(Se^{79}) + \lambda_s(Se^{79})].$$
where \( \lambda_n \) and \( \lambda_b \) are the neutron-capture and beta-decay rates respectively. The coefficient 0.92 appears because in the loop the nuclei have to pass through Br\(^{80} \) which beta decays only 92\% of the time. On dividing these two equations and substituting in terms of the half-lives \( t_n \) and \( t_b \) instead of the rates, we find that

\[
\frac{t_n(Se^{37}) + t_b(Se^{37})}{t_n(Se^{37})} = 0.92 \frac{t_n(Kr^{80}) N(Kr^{80})}{t_n(Se^{37})}.
\]

Now \( \sigma(Kr^{80}) \approx 2 \sigma(Kr^{85}) \) so that \( \lambda_n(Kr^{80}) \approx 2 \lambda_n(Kr^{85}) \) and \( t_n(Kr^{80}) \approx 2 t_n(Kr^{85}) \). [In the appendix we put \( \sigma(Kr^{80}) \approx \sigma(Kr^{85}) \) and do not in general take into account the factor of \( \approx 2 \) which is a result of the fact that the light neutron-poorer isotopes capture neutrons more readily than the heavier ones.] The abundance ratio \( N(Kr^{80})/N(Kr^{85}) \) is given in the appendix. Thus substituting these values on the right-hand side of the equation we find for \( Se^{37} \) that

\[
t_n/t_b = 1/1.37,
\]

and, since \( t_b = 7 \times 10^4 \) years, \( t_n = 5.1 \times 10^4 \) years. Now this half-life for neutron capture is determined in a region where \( \sigma_n = 360 \) mb, so that in the region near iron where this neutron chain begins and where \( \sigma = 30 \) mb the effective half-life is \( 5.1 \times 10^4/360/30 \approx 6 \times 10^4 \) years. The total time demanded to build up the isotopes from iron if this were characteristic of the whole curve would be \( 10^8-10^9 \) years since there are many captures involved, but most of these take place on nuclei with larger \( \sigma \) values than that for iron. We show later that this long time-scale for the \( s \) process is probably the result of a paucity of neutrons—probably only about five neutrons per iron nucleus are made available. This is reflected in the tendency for the \( \sigma N \) product to decrease rather rapidly as \( A \) increases.

The second indicator for an \( s \) process time-scale comes from consideration of the isotopes of gadolinium at \( A = 152 \) and 154. Of these isotopes Gd\(^{152} \) can only be made by the \( s \) process. On the other hand the majority of Gd\(^{152} \) is probably made in the \( p \) process (Sec. IX), since its abundance fits in well with the run of abundances of the isotopes made by this process in this region of the curve. We shall arbitrarily suppose that of the total abundance of Gd\(^{152} \) of 0.00137 about 0.001 is made by the \( p \) process (cf. Fig. IX.1) and the remainder by the \( s \) process, since the following argument is in no way critically dependent on this division. The small amount made in the \( s \) process must be due to the fact that Gd\(^{152} \) is mostly by-passed in the \( s \)-process chain. Reference to the appendix shows that the branching which does this cannot take place at Eu\(^{152} \) since in 28\% of the cases Eu\(^{152} \) will decay to Gd\(^{152} \) while in 78\% of the cases isomeric Eu\(^{152m} \) will decay to Gd\(^{152} \). The combination of these two decays shows that in 36\% of the cases Eu\(^{152} \) will decay to Gd\(^{152} \) if the production ratio Eu\(^{152}/Eu^{152m} \approx 5 \). Thus the branching responsible for this by-pass must take place at Sm\(^{151} \) which has a half-life for beta decay of 80 years and which must mainly capture a neutron before beta decay. Thus we must have that

\[
\frac{t_n(Sm^{151})}{t_b(Sm^{151})} = \frac{N'(Gd^{152})}{N(Gd^{154})},
\]

where \( N'(Gd^{152}) \) is the abundance of Gd\(^{152} \) estimated on the supposition that 36\% of the decays from Eu\(^{152} \) give an abundance of 0.00037. That is, \( N'(Gd^{152}) = N(Gd^{154}) = 0.00037/0.36 \approx 0.001 \), and \( N(Gd^{154}) \) is the abundance of this isotope given in the appendix. Thus substituting for \( N'(Gd^{152}) \), \( N(Gd^{154}) \), and \( t_b(Sm^{151}) \) we find that \( t_n(Sm^{151}) \approx 5 \) years. Using the ratio of the \( \sigma's \) in this region as compared with iron we find that near iron \( t_n(Sm^{151}) \approx 10^4 \) years. The mean value of \( \sigma \) near \( A = 150 \) is about 1000 mb whereas \( \sigma(Sm^{151}) = 1440 \) mb. Thus the mean \( t_n \) near \( A = 150 \) is of the order of 10 years and the total time for the capture of \( \approx 200 \) neutrons is about \( 2 \times 10^4 \) years. We show later that this shorter of the two time-scales of the \( s \) process is probably due to the fact that plenty of neutrons are available and that the "steady flow" concept can be used. In this case the \( \sigma N \) product remains approximately constant.

\section{6 Time-Scales of the Various Processes}

Several alternative processes which may synthesize deuterium, lithium, beryllium, and boron are described in Sec. X. Depending on which of these is the most satisfactory, the time-scale for synthesis may range from a few seconds if a supernova origin is assumed to \( \approx 10^7 \) years if the surfaces of active stars are considered probable. The time-scale here is important only in as far as it may allow some predictions of the probable distribution of these elements in the solar system and the Galaxy.

In Fig. II.4 we give a schematic diagram showing where the various synthesizing processes take place in a plot of the internal stellar temperature against the
time that the star lives in each temperature range. The estimated time-scales associated with each of these processes are also given very approximately.

III. HYDROGEN BURNING, HELIUM BURNING, THE α PROCESS, AND NEUTRON PRODUCTION

This section and the sections to follow are devoted to detailed elaboration and discussion of the different physical processes introduced in Sec. II. These sections treat quantitatively experimental and theoretical evaluations of the cross sections and reaction rates of the nuclear processes involved in energy generation and element synthesis in stars. The material supplements and extends that published in a series of articles in 1954 (Fo54, Bo54, Ho54), in 1955 (Fo55, Fo55a), and in 1956 (Bu56). In the first part of this section we give a discussion of the relations between nuclear cross sections and nuclear reaction rates in stellar interiors and of the notation used in this and the following sections.

A. Cross-Section Factor and Reaction Rates

The experimental results to be discussed will be used to derive the numerical value of the nuclear cross-section factor for a charged particle reaction defined by

\[ S = \sigma(E)E \exp(31.28Z_1Z_2AE^{-1}) \text{ kev barns} \]

where \( \sigma(E) \) is the cross section in barns (\( 10^{-24} \text{ cm}^2 \)) measured at the center-of-mass energy \( E \) in kev. The charges of the interacting particles are \( Z_1 \) and \( Z_2 \) in units of the proton charge and \( A = A_1A_2/(A_1+A_2) \) is their reduced mass in atomic mass units. \( S \) is measured in the center-of-mass system. From measurements made in the laboratory system with incident particle energy \( E_1 \) and with target nuclei at rest, the quantity \( S \) is given by

\[ S = \sigma(E_1)E_1 \frac{A_0}{A_1+4} \exp(31.28Z_1Z_2AE_1^{-1}) \text{ kev barns}. \]

For a nonresonant or off-resonant reaction \( S \) is a slowly varying function of the energy \( E \). Methods for extrapolating to the effective thermal energy \( E_\text{\text{th}} \) in stellar interiors have been given by numerous authors (Sa52a, Fo54, Sa55, Ma57). The effective thermal energy at temperature \( T \) is

\[ E_\text{\text{th}} = 1.220(Z_1^2Z_2^2AT_\text{\text{th}}^2)^{1/4} \text{ kev barns}, \]

where \( T_\text{\text{th}} \) is the temperature measured in units of \( 10^9 \) °K. The width of the effective range of thermal energy is

\[ \Delta E_\text{\text{th}} = 0.75(Z_1^2Z_2^2AT_\text{\text{th}}^2)^{1/4} \text{ kev.} \]

The mean reaction rate of a thermonuclear process may be expressed as

\[ P = \rho \tau = n_1n_0\langle \sigma v \rangle_{\text{\text{th}}} = 3.63 \times 10^{23} \rho \langle \sigma v \rangle_{\text{\text{th}}} A_1A_2/4 \text{ reactions cm}^{-2} \text{ sec}^{-1}, \]

where \( n_1 \) and \( n_0 \) are the number densities of the interacting particles per cm\(^3\) and \( \langle \sigma v \rangle_{\text{\text{th}}} \) is the average of the cross section multiplied by the velocity in cm\(^3\) sec\(^{-1}\). The quantity \( \tau \) is the reaction rate per gram per second. The quantities \( x_1 \) and \( x_0 \) are the amounts of the interacting nuclei expressed as fractions by weight. In terms of \( S_\text{\text{th}} = S(E_\text{\text{th}}) \) kev barns, it is found for a nonresonant process that

\[ P = 7.20 \times 10^{-18}n_1n_0f_S(AZ_1Z_2)^{-1}t_\text{\text{th}} \]

\[ = 2.62 \times 10^{29} \frac{x_1x_0}{A_1A_2} f_S(AZ_1Z_2)^{-1}t_\text{\text{th}} \text{ reactions cm}^{-3} \text{ sec}^{-1}, \]

where \( S_\text{\text{th}} \) is in kev barns and

\[ \tau = 42.48 \left( Z_1^2Z_2^2 \frac{A}{T_\text{\text{th}}} \right)^{1/4} \text{ sec}^{-1}, \]

(this \( \tau \) is not to be confused with the mean lifetime of the interacting particles which will always be accompanied by appropriate subscripts, etc.). The term \( f_S \) is the electron screening or shielding factor discussed by Salpeter (Sa54), evaluated at \( E_\text{\text{th}} \). The cross-section factor \( S_\text{\text{th}} \) as customarily calculated does not include allowance for electron screening.

The mean lifetime of the nuclei of type 0 for the interaction with nuclei of type 1 is given by

\[ \tau(0) = \rho(0) = \rho \tau \]

\[ = 4.34 \times 10^{29} \frac{t_\text{\text{th}}}{A_1} f_S(AZ_1Z_2)^{-1} \text{ reactions cm}^{-3} \text{ sec}^{-1}, \]

\[ = 7.83 \times 10^{29} \frac{t_\text{\text{th}}}{A_1} f_S \left( \frac{Z_1Z_2}{AT_\text{\text{th}}} \right)^{1/4} \text{ sec}^{-1}, \]

where \( n_0 \) is the number of nuclei of type 0 consumed in each reaction. The quantity \( \rho(0) \) is the mean reaction rate per nucleus of type 0. If nuclei of type 0 are regenerated in a cycle of reactions then \( \tau(0) \) becomes the mean cycle time for nuclei of type 0.

The most satisfactory procedure for determining \( S_\text{\text{th}} \) is to make experimental observations on cross sections over a range of energies not too large compared to \( E_\text{\text{th}} \). The cross-section factor, \( S \), can then be plotted as a function of \( E \) and an appropriate extrapolation to find \( S_\text{\text{th}} \) can be made. This is not always possible and computational procedures for several frequently occurring cases will now be given.

(i) For the case in which \( S_\text{\text{th}} \) is to be calculated from the experimentally determined parameters of a reso-
nance at $E_r$ which falls outside the range $E_o \pm 2\Delta E_o$, then

$$\sigma = \pi \lambda^2 \alpha \frac{\Gamma_1 \Gamma_2}{(E-E_o)^2 + \Gamma^2/4},$$

and

$$S_o = 3.10 \times 10^{-18} \frac{1 - \alpha E_o}{E} \frac{\omega \Gamma E_R}{A K_{2l+1}(x)} \frac{(E-E_o)^2 + \Gamma^2/4}{\text{kev barns}}.$$

The corrections for level shift effects are given by Marion and Fowler (Ma57). The various quantities which enter into these expressions are:

- $\lambda$: reduced De Broglie wavelength of interacting particles 0 and 1
  $$\lambda = \left( \frac{\hbar^2}{2ME} \right)^{1/2} = 144\left( \frac{AE}{E} \right)^{-1} \text{ fermis}$$
  $$R/\lambda = (E/E_0)^{1/2},$$
  $$R = \text{interaction radius} = 1.44\left( \frac{A_1 + A_d}{A} \right) \text{ fermis}$$
- $\theta^2$: dimensionless reduced width for the interaction of particle 1 with particle 0, derivable from the observed width, $\Gamma_1$
  $$\Gamma_1 = \text{width for re-emission of particle 1 with particle 0} = 60a^2 \left( \frac{2MR^3}{E_0} \right)^{1/2} P_1$$
  $$\omega = \text{statistical factor} = \left( 2J + 1 \right) / \left( 2J_{1l} + 1 \right) \left( 2J_{0l} + 1 \right)$$
- $\Gamma_{2l+1}$: angular momentum of resonant state
  $$\Gamma_{2l+1} = \text{angular momentum of particle 1}$$
- $J_{2l}$: angular momentum of resonant state
  $$J_{2l} = \text{angular momentum of particle 0}$$
- $x = 2(E_c/E_0)^{1/2} = 0.525(4Z_1 Z_0 R)^{1/2}$
- $E_R = \left[ 1 / (l+1) \right] \times \text{centrifugal barrier height}$
  $$= \hbar^2 \left( 2MR^2 \right)^{1/2} \times 10^{3} \text{ kev}$$
- $E_R = \text{Coulomb barrier height} = Z_1 Z_0 R^2 / R$$
  $$= (1.44 Z_1 Z_0 R) / 10^{3} \text{ kev}.$$

For large $x$ (high Coulomb barrier):

$$K_{2l+1}(x) \approx (\pi/2x)^{1/2} \exp \left[ -x + \frac{2(l+1)^2}{x} \right]$$

$$P_l \approx E_l^{-1/2} \left( 1 - \alpha l E \right) \exp \left[ -31.28 Z_1 Z_0 A^{1/4} E^{-1} \right]$$

$$P_l / E_0 \approx \exp \left[ -2(l+1)(E_0/E) \right]$$

$$\alpha_{l=0} = \frac{2}{3} \left( E_c / E_R \right)^{1/2} \approx 10^{-3} (\text{kev})^{-1}.$$

In principle the cross sections and cross-section factors given above must be summed over all values of the orbital angular momentum, $l$th. The expressions given apply to any one specified $l$-wave interaction. Because of the low energies involved in nuclear collisions in stellar interiors, $s$-wave ($l=0$) interactions and in some cases $p$-wave ($l=1$) interactions are usually the only ones which need to be taken into account. However, whenever selection rules or destructive interference effects reduce the low $l$-value interactions to zero, then higher $l$ values must be considered. In addition, whenever any particular $l$-wave interaction is resonant, see (iv), then it must be taken into account. Resonant cross sections can be quite large even for high $l$ values.

(ii) For the case in which the cross section is an average over resonances of the same spin and parity with mean width $\Gamma$, spaced by an average energy interval equal to $D$, then averaging over the expression for $\sigma$ given in (i) yields

$$\bar{\sigma} = 2\pi \lambda (2l+1) \Gamma_{1l} \Gamma_{2l} / D \Gamma.$$

In general, each $l$ wave can form several compound systems of different spins but the same parity. In employing the above approximate relation with the simple statistical factor $(2l+1)$ we assume that these different systems have the same mean widths, $\Gamma$, and the same mean level separations, $D$.

With the foregoing expression for $\bar{\sigma}$, one finds, neglecting $E_n$, 

$$S_o = 1.94 \times 10^{-17} \frac{\theta^2}{A K_{2l+1}(x)} \frac{2l+1}{\Delta E_x} \frac{\Gamma_{2l} E_R}{\Gamma \Delta D} \text{ kev barns}.$$

This will prove to be useful for order of magnitude estimates for interactions of charged particles with heavy nuclei ($Z \gtrsim 10$). Blatt and Weisskopf (Bl52) have given a general relation between the particle decay widths, $\Gamma_1 = \theta^2$, for a given system of resonances and
their mean energy separation. This can be written

\[ \theta^2 = \frac{\lambda_0}{3\pi R E_R} \beta(A_0). \]

The function \( \beta(A_0) \) varies between \( \sim 0.3 \) and 3 for all except a few light and intermediate weight nuclei (Fe54, We57a) and we will set it equal to unity for order of magnitude estimates in what follows. The quantity \( \lambda_0 = 0.7 \times 10^{-13} \text{ cm} \) is the characteristic reduced De Broglie wavelength of low-energy nucleons once inside nuclei.

We now have, for \( R \) in fermis,

\[ S_\nu \sim 1.4 \times 10^4 \frac{2I+1}{\pi K_{214}^2(x) \Gamma} \text{ kev barns}. \]

For \( l=0 \) and large \( x \) we then have

\[ S_\nu(\nu=0) \sim 1.8 \times 10^6 \frac{1}{\pi K_{214}^2(x) \Gamma} \left( \frac{E_C}{E_R} \right)^{1/4} \exp \left[ -\frac{4}{E_R} \left( \frac{E_C}{E_R} \right)^{1/4} \right] \text{ kev barns}. \]

In general for \( (\rho, \gamma) \) reactions on the light nuclei (\( Z = 6 \) to 12) one finds \( S_\nu \sim 1 \) to 100 kev barns. For the \( (\rho, \alpha) \) reactions, \( S_\nu \sim 1 \) to 10 kev barns. On substitution into the expression for the mean lifetime, and setting \( \nu_0 = 1, f_0 = 1, l = 0 \), it is found that

\[ \frac{1}{\tau_1(0)} = 3.8 \times 10^6 \frac{1}{\pi K_{214}^2(x) \Gamma} \left( \frac{E_C}{E_R} \right)^{1/4} \exp \left[ -\frac{4}{E_R} \left( \frac{E_C}{E_R} \right)^{1/4} \right] \sec^{-1}. \]

In cases where this expression is applicable, e.g., \( (\rho, \gamma) \) or \( (\alpha, n) \) reactions on heavy nuclei (\( Z \geq 10 \)) one will in general have \( \Gamma = \Gamma_2 \gg \Gamma_1 \).

(iii) In the case of light nuclei, the interaction energy may fall in the flat minimum in between resonances. For this nonresonant case, order of magnitude estimates may be made by summing the contributions of all resonances of a given type with width \( \Gamma \) spaced by an average energy interval equal to \( D \). The nonresonant cross section for a given \( l \)-wave interaction is

\[ \sigma_{nl} = \pi \lambda^2 (2I+1)(\pi^2 + 8) \Gamma_1 \Gamma_2 / D^3. \]

This expression assumes that all except the two nearest resonances have random phases, that is, that their contributions to the cross section can be simply added without compounding amplitudes. The factor \( (\pi^2 + 8) \) is to be used if the two nearest resonances interfere constructively; the factor \( (\pi^2 - 8) \) if the two nearest resonances interfere destructively. In fact, it is possible for the cross section for a given \( l \)-wave to go to zero if all of the contributing states interfere to give zero reaction amplitude. In this case the next highest \( l \)-wave must be considered. On the average, for order of magnitude estimates, we can assume that all resonances, including the two nearest ones, are randomly phased in which case the factor \( \pi^2 \pm 8 \) becomes \( \pi^2 \). The expression for \( \sigma_{nl} \) is then the same as \( \sigma \) in (ii) with \( \Gamma \) replaced by \( 2D/\pi^2 \).

With this substitution, expressions for \( S_\nu \) and \( 1/\tau_1(0) \) given in (ii) can be employed. It will be noted, however, that in general \( \Gamma_2 < D \) so that an approximation similar to \( \Gamma_2 \sim \Gamma \) is not valid in this case. For the light nuclei \( D \sim 1 \) to 10 MeV, whereas for radiative processes \( \Gamma_2 = \Gamma_1 \sim 0.1 \) to 100 ev so that \( 10^{-7} \ll \Gamma_2 / D < 10^{-4} \). For particle emission \( \Gamma_2 / D \) may be as high as 0.1.

(iv) When a resonance \( E_r \) falls within the range \( \sim E_o \pm 2\Delta E_o \) the resonant cross section factor \( S_r \), derivable from the resonant cross section \( \sigma_r \), must be employed. When resonant, even fairly large \( l \)-wave interactions must be considered. We have

\[ \sigma_r = 4\pi \lambda^2 \omega^2 \Gamma_2 / D^3. \]

We will consider two limiting cases,

\[ \sigma_r \rightarrow 4\pi \lambda^2 \omega^2 \Gamma_2 / D^3 \text{ for } \Gamma = \Gamma_2 > \Gamma_1, \]

\[ \sigma_r \rightarrow 4\pi \lambda^2 \omega^2 \Gamma_2 / \Gamma_1 \text{ for } \Gamma = \Gamma_2 \gg \Gamma_1. \]

The cross-section factor is

\[ S_r = 1.24 \times 10^6 \frac{\theta^2}{A K_{214}^2(x) \Gamma^2} \frac{1 - \alpha_1 E_r}{\omega^2 \Gamma_2} \text{ kev barns}. \]

The two limiting cases become

\[ S_r \rightarrow 1.24 \times 10^6 \frac{\theta^2}{A K_{214}^2(x) \Gamma^2} \frac{1 - \alpha_1 E_r}{\omega^2 \Gamma_2} \text{ kev barns} \]

\[ S_r \rightarrow 2.63 \times 10^3 \frac{\omega^2 \Gamma_2}{\omega \Gamma_1} \text{ kev barns} \]

The resonant reaction rate is for \( \Gamma \) in kev and \( S_r \) in kev barns;

\[ P = 3.08 \times 10^{-15} \frac{f_r}{AT_0^3} s^{-1} \text{ cm}^{-3} \text{ sec}^{-1} \]

\[ = 1.12 \times 10^{10} \frac{\lambda^2}{A_1 A_0 (AT_0)^3} S_r \Gamma_2 \exp (-\frac{1}{2} \Gamma_2) \text{ reactions cm}^{-3} \text{ sec}^{-1} \]

\[ -1.39 \times 10^{10} \frac{\lambda^2}{A_1 A_0 (AT_0)^3} \frac{\omega E_R \Gamma_2}{\Gamma_0} \exp (-\frac{1}{2} \Gamma_2) \text{ reactions cm}^{-3} \text{ sec}^{-1} \]

\[ -2.94 \times 10^{10} \frac{\lambda^2}{A_1 A_0 (AT_0)^3} \exp (-11.61 E_r / T_0) \text{ reactions cm}^{-3} \text{ sec}^{-1} \]

for \( \Gamma = \Gamma_2 > \Gamma_1 \).
In the preceding expressions

$$\tau_r = 51.28 Z \alpha Z e A \Gamma e^{-1} + 11.61 E_r / T_0$$

and $f_1$ is the electron shielding factor evaluated at $E_r$.

The mean reaction time of nuclei of type 0 is

$$\frac{1}{\tau_1(0)} = 1.86 \times 10^{10} \left( \frac{\rho_{11}}{A} \right) \Gamma e^{-r} \text{ sec}^{-1}$$

$$- 2.31 \times 10^{10} \left( \frac{\rho_{11}}{A} \right) \exp \left( -11.61 E_r / T_0 \right)$$

$$- 0.49 \times 10^{10} \left( \frac{\rho_{11}}{A} \right) \exp \left( -11.61 E_r / T_0 \right)$$

for $\Gamma = \Gamma_2 \gg \Gamma_1$

$$- 0.49 \times 10^{10} \left( \frac{\rho_{11}}{A} \right) \exp \left( -11.61 E_r / T_0 \right)$$

for $\Gamma = \Gamma_1 \gg \Gamma_2$.

**B. Pure Hydrogen Burning**

The point of view that element synthesis begins with pure hydrogen (primordial or continuously produced) condensed in stars is based on the existence of the so-called direct $pp$ chain of reactions by which hydrogen is converted into helium. This chain is initiated by the direct $pp$ reaction

$$p + p \rightarrow d + \beta^+ + \nu_e + 0.421 \text{ Mev},$$

which has good theoretical foundations but which has not yet been observed experimentally in the laboratory because of its extremely low cross section even at relatively high interaction energies. In the above equation and in what follows we use $\nu_e$ for neutrinos emitted with positrons, $\beta^+$, and $\nu_e$ for antineutrinos emitted with electrons, $\beta^-$. We use nuclear rather than atomic mass differences in expressing the $Q$ values of all reactions. There is of course practically no difference in atomic and nuclear $Q$ values when positrons or electrons are not involved.

The calculated cross section for the $pp$ reaction is $10^{-17}$ cm$^2$ = $10^{-23}$ barn at 1-Mev laboratory energy. This is much too small for detection with currently available techniques. The yield when a thick hydrogen target is employed can be estimated to be $5 \times 10^{-28}$ reaction per incident proton or approximately one reaction per $3 \times 10^8$ amp sec corresponding at 10$^8$ volts to $3 \times 10^4$ w sec. Thus only one reaction per 10 Megawatt years of bombardment can be expected. In an earlier paper (Fo54) it was indicated that the cross section would reach a maximum value at 100 kev. This is not the case if account is taken of the rapid increase in beta-decay probability with increasing beta-decay energy and hence with increasing incident proton energy. At the present time, however, it is not possible to obtain within many orders of magnitude, the necessary amounts of bombardment even at 1 Mev or above. At 100 Mev the cross section becomes $3 \times 10^{-16}$ barn but very pure hydrogen targets would be necessary to avoid positron production in heavy nuclei as contaminants.

At still higher energy, energy production becomes a complication.

**1. Rate of $pp$ Reaction**

Definitive theoretical calculations on the rate of the $pp$ processes were given by Salpeter (Sa52a). These calculations accurately took into account the well-determined phenomenological parameters of the neutron-proton and proton-proton interactions and showed that the main uncertainty in the cross section lay at that time in the uncertainty of the Gamow-Teller beta-decay constant for which the following value was given:

$$g = 7.5 \pm 1.5 \times 10^{-4} \text{ sec}^{-1} \text{ (Sa52a)}.$$
which is 29\% less than the value given by Salpeter (Sa52a) and used by Bosman-Crespin et al. (Bo54) in numerical calculations. These numerical values should thus be reduced by 29\%. The over-all probable error, taking other sources of error into account, can be conservatively estimated as 10\%. The cross-section factor becomes

$$S = 28.5(1 + 0.008E) \times 10^{-20} \text{ kev barn} \pm 10\%$$

for \( E \) in kev or

$$S_e = 28.5(1 + 0.008T_e^4) \times 10^{-20} \text{ kev barn} \pm 10\%.$$ 

The correction term in \( E \) or \( T_e^4 \) arises from the dependence on energy of the beta-decay transition probability, and in addition from small corrections to the barrier penetrability when a nonzero interaction radius is assumed. It is accurate only for \( E \leq 100 \) kev or \( T_e^4 \leq 10^6 \) degrees. At laboratory bombarding energies for \( E \geq 1 \) Mev, \( S \sim 2E^5 \times 10^{-24} \text{ Mev barn, } \sigma \sim 3E_3^{4.5} \times 10^{-24} \text{ barn.} \)

In the following discussion it is convenient to use a quantity \( \alpha_{pp} \), which is involved in the reaction rate and is given by

$$\alpha_{pp} = \rho_{pp} T_s^{-4}(1 + 0.0127T_s^4 + 0.008T_s^6 + 0.00065T_s^8)$$

$$\times \exp(-33.804T_s^{-1}) \pm 10\%$$

\( \sim 2.0 \times 10^{-7} \rho_{pp} \left( \frac{T_s}{15} \right)^{3.95} \) near \( T_s = 15 \).

The electron screening factor, \( f_{pp} \), can be taken as approximately equal to unity for \( T_s \geq 8 \). The quantity \( \rho \) is the density and \( x_H \) is the concentration by mass of hydrogen. The correction terms in \( T_s^4 \) and \( T_s \) arise in integrating over all the interaction energies. The first has been discussed by Salpeter (Sa52a) and the second comes when the energy dependence of \( S \) is included to first order in the integration. The reaction rate in reactions per gram of material per second is \( \alpha_{pp} x_H^2 \) multiplied by a numerical factor derivable from \( S_e, \) viz.,

$$r_{pp} = 0.833 \alpha_{pp} x_H^2 \times 10^{11} \text{ reactions g}^{-1} \text{ sec}^{-1}$$

while the mean reaction rate per proton with two protons consumed per reaction is

$$p_{pp}(H) = \int \frac{1}{x_H} \frac{dx_H}{dt} = 2.78(\alpha_{pp} x_H) \times 10^{-19} \text{ sec}^{-1}.$$ 

If the \( pp \) chain is completed then the energy release in the over-all process, 4H\( \rightarrow \)He\(^4\), is 26.22 Mev = 4.201 \( \times 10^{10} \) ergs, using the atomic masses of Wapstra (Wa55). This value excludes the 2\% neutrino energy loss. The expression for the energy generation, \( \epsilon_{pp} \), is then given by

$$\epsilon_{pp} = 0.5 \times 4.201 \times 10^{-7} \rho_{pp} \text{ ergs}$$

or

$$\epsilon_{pp} = 1.75 \alpha_{pp} x_H^2 \times 10^6 \text{ ergs g}^{-1} \text{ sec}^{-1} \ (4\text{H} \rightarrow \text{He}^4).$$

This energy generation as a function of temperature is tabulated in Table III,1 and is indicated graphically in

![Diagram](image)

**Fig. III,1. Energy generation in ergs g\(^{-1}\) sec\(^{-1}\) as a function of temperature in the \( pp \) chain and the \( CN \) cycle. The ordinates give loge directly for \( x_{pp} \rho_{pp} = 100 \ g/cm^3, \rho_{pp} x_H = 1 \ g/cm^3 \) or \( x_{pp} x_H = 1 \ g/cm^3 \). They are thus appropriate for \( x_{pp} = x(N^3) \) or \( x_0 = x(C^3) \). If \( 0.8x_{pp} x_H \) is resonant the \( CN \) cycle rate is determined by \( CH(p, \gamma) \) and \( x_{pp} \approx 0.8x_N \). If \( N^3(p, \gamma) \) is nonresonant, it determines the \( CN \) cycle rate and \( x_N = x_C. \)**
Table III.2. Mean lifetimes or cycle times in hydrogen burning (years). *(see text for details)*

<table>
<thead>
<tr>
<th>( T ) (10^6 K)</th>
<th>( \log \tau_{\beta\beta}(I) )</th>
<th>( \log \tau_{\beta\beta}(II) )</th>
<th>( \log \tau_{\beta\beta}(III) )</th>
<th>( \log (\tau_{\beta\beta}/100) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>+19.0</td>
<td>+22.5</td>
<td>+12.09</td>
<td>+16.5</td>
</tr>
<tr>
<td>10</td>
<td>+12.0</td>
<td>14.7</td>
<td>10.51</td>
<td>9.53</td>
</tr>
<tr>
<td>15</td>
<td>8.63</td>
<td>11.0</td>
<td>9.76</td>
<td>6.15</td>
</tr>
<tr>
<td>20</td>
<td>6.30</td>
<td>8.61</td>
<td>9.29</td>
<td>4.02</td>
</tr>
<tr>
<td>30</td>
<td>3.83</td>
<td>5.65</td>
<td>8.72</td>
<td>1.35</td>
</tr>
<tr>
<td>40</td>
<td>2.15</td>
<td>3.79</td>
<td>8.36</td>
<td>0.00</td>
</tr>
<tr>
<td>50</td>
<td>0.96</td>
<td>2.47</td>
<td>8.11</td>
<td>1.52</td>
</tr>
<tr>
<td>70</td>
<td>−0.72</td>
<td>0.56</td>
<td>7.76</td>
<td>1.32</td>
</tr>
<tr>
<td>100</td>
<td>−2.22</td>
<td>−1.00</td>
<td>7.44</td>
<td>−4.7</td>
</tr>
</tbody>
</table>

The mean lifetime of hydrogen in the \( pp \) chain becomes

\[
\tau_{pp}(H) = 1.14 (\varepsilon_{pp}/E_{pp})^{1/10} \text{ years} \\
= 1.99 (x_{II}/\varepsilon_{pp}) \times 10^{11} \text{ years} \\
= \frac{9.9 \times 10^{10}}{E_{pp}} \text{ years,}
\]

where \( E_{pp} \) is the quantity tabulated by Bosman-Crespin et al. (Bo54) decrease by 29\%. It will be recalled that \( \rho \times 10^{-10} \text{ g/cm}^3 \) at the center of the sun and similar stars. Hydrogen lifetimes at relevant temperatures are given in Table III.2 and Fig. III.2.

The recent detection (Re53, Co56a) of the free antineutrino through absorption by hydrogen in the reaction

\[
\nu_e + p \rightarrow p + e^- + \nu_e \rightarrow n + \beta^- + 1.804 \text{ MeV}
\]

has led to increased confidence in Fermi's neutrino theory of beta decay, and thus in the theoretical calculations. The rate of the \( pp \) reaction which has just been described. In addition, the interesting possibility arises of investigating antineutrino capture in deuterium in processes which would be inverse to the direct \( pp \) reaction. Since antineutrinos are produced in the negative beta decay in a neutron reactor, observation of the following reaction

\[
\nu_e + d \rightarrow p + p + \nu_e \rightarrow n + \beta^- + 1.443 \text{ MeV},
\]

which is directly inverse to the \( pp \) reaction, would require that antineutrinos and neutrinos be identical. Recent experiments (Aw56), which show that the half-lives of Cs\( ^{138} \) and Zr\( ^{98} \) to double beta decay are \( \geq 10^{18} \) years, indicate that this is not the case. In addition, if \( \nu_e \neq \nu_e \), the reaction \( \beta^+ + \nu_e \rightarrow \beta^+ + \nu_e \) would not be expected to take place and this has been shown to be the case (Da56). Also, the \( \nu_e + d \) process would be very difficult to detect over background because the energy given to the protons and to the electron will be dissipated promptly in electronic collisions. The process will not have a distinguishing signature as does the antineutrino absorption by protons, the slowing down of the neutron gives an observable delay up to about as 17 \( \mu \)sec (Co56a) between the positron annihilation energy release and the neutron capture energy release.

On the other hand, the "mirror-inverse" process to the \( pp \) reaction, which is

\[
\nu_e + d \rightarrow n + n + \beta^+ \rightarrow 4.029 \text{ MeV},
\]

can be induced by reactor antineutrinos of sufficient energy and it will have a signature even more distinctive than that for the absorption in light hydrogen. Only a
small fraction of the reactor antineutrinos have energies above the threshold for this process and thus it will be much more difficult to observe than the absorption by protons. In fact, in the current experiments by Cowan et al. (Co56a) deuterium is used in place of part of the hydrogen in order to show that the observed events are directly proportional to the proton concentration. While this paper has been in preparation, two papers have appeared on the antineutrino disintegration of the deuteron (Mu57, We57) which give the effective cross sections for reactor neutrinos as $\sigma(\nu_e+p)=6\times10^{-44}$ cm$^2$ and $\sigma(\nu_e+d)=2\times10^{-44}$ cm$^2$. These must be multiplied by a factor of 2 in order to account for parity-nonconserving beta transitions. In any case the deuteron disintegration cross section is only 1/30 of that for the proton. However, if the mirror inverse process can ultimately be observed, the observation would constitute fundamental experimental confirmation of the very general ideas underlying the $pp$ process. Note added in proof.—Cowan and Reines (Co57a) have not been successful in detecting the $\nu_e+d$ reaction but their experimental upper limit of $\sigma(\nu_e+d)=4\times10^{-44}$ cm$^2$ does not exclude the theoretically expected value given above.

(2) Other Reactions in the $pp$ Chain

New measurements of the cross section for $^3D^0(p,\gamma)$ have been reported (Gr55) since the review by Fowler (Fo54), but these new values do not necessitate any significant change in the value of the cross-section factor given there ($S_0=8\times10^{-4}$ kev barn). The final results for the $He^4(He^4,2p)He^4$ cross section have been published (Go54). The value of $2.5\times10^{-9}$ barn at $E_{lab}=200$ kev or $E_{m}=100$ kev yields a cross section factor equal to 1200 kev barn. Fairly substantial corrections for nonzero radius of interaction in the barrier penetrability energy dependence and for electron shielding effects will increase this to an effective value, $S_0=2000$ kev barns, at relevant stellar energies.

C. Pure Helium Burning

When hydrogen burning in a star’s main-sequence stage leads eventually to hydrogen exhaustion, a helium core remains at the star’s center. It has been suggested (Sa52, Sa53, Op51, Op54) that the fusion of helium plays an important role in energy generation and element synthesis in the red-giant stage of the star’s evolution. The fusion occurs through the processes

$$He^4+He^4\rightarrow Be^8$$

$$Be^8+He^4\rightarrow C^{12}+\gamma$$

or, in a more condensed notation, through

$$3He\rightarrow C^{12}+\gamma.$$

We refer to this as the $3\alpha$ reaction. These processes are believed to occur at a late stage of the red giant evolution in which the hydrogen in the central core has been largely converted into helium, and in which gravitational contraction (Ho55) has raised the central temperature to $\sim10^8$ degrees, and the density to $\sim10^6$ g/cc. Under these conditions, as shown by Salpeter, an equilibrium ratio of $Be^8$ to $He^4$ nuclei equal to $\sim10^{-9}$ is established. This conclusion followed from experimental measurements (He48, He49, To49, Wh41) which established the fact that $Be^8$ was unstable to disintegration into two alpha particles but only by 95 kev with an uncertainty of about 5 kev.

(1) Rate of the $3\alpha$ Reaction

Even though very small, the equilibrium concentration of $Be^8$ is sufficient to lead to considerable production of $C^{12}$ through radiative alpha-particle capture by the $Be^8$, and of $O^{16}$, $Ne^{20}$, etc., by succeeding alpha-particle captures. Salpeter’s calculations, in which resonance effects due to the $Be^8$ ground state were taken into account, indicated a rate for the helium-burning considerably greater at a given pressure and temperature than that previously calculated by $\acute{O}$pik using nonresonant reaction rates (Op54). Detailed consideration of the reaction rates and of the resulting relative abundances of $He^1$, $C^{12}$, and $O^{16}$ led Hoyle (Ho54) to the prediction that the foregoing second reaction, in which $C^{12}$ is produced, must exhibit resonance within the range of energies at which the interaction between $Be^8$ and $He^4$ effectivley occurs. Hoyle’s predicted value for the resonance energy was 0.33 Mev, corresponding to an excited state in $C^{12}$ at 7.70 Mev.

It recently has been shown (Fo56, Co57) through the studies of the alpha emission following the beta decay of $B^{10}$ to the second excited state of $C^{12}$, that this state does decay into three alpha particles through an intermediate stage involving $He^4$ and the ground state of $Be^8$. The general reversibility of nuclear reactions thus leads one to expect that $C^{12}$ will be produced in the Salpeter reactions given above and that the second excited state of $C^{12}$ does indeed play a dominant role in the synthesis of elements from helium, as predicted by Hoyle.

The experiments reported (Co57) show that $Q(He^8\rightarrow 2He^4)=93.7\pm0.9$ kev, that the excitation energy of $C^{12*}$ is $Q(C^{12*}\rightarrow C^{12})=7.653\pm0.008$ Mev, that $Q(C^{12*}\rightarrow He^8\rightarrow He^4)=278\pm4$ kev, $Q(C^{12*}\rightarrow 3He^8)=372\pm4$ kev, and that the most probable spin and parity assignments for $C^{12*}$ are $^2I^+$. The state must have either even spin with even parity or odd spin with odd parity. Otherwise, by the conservation of angular momentum and parity, it cannot decay into three alpha particles through the ground state of $Be^8$. Since the $Be^8$ is in equilibrium, the reaction rate for the conversion of helium into carbon can be calculated simply from the $3He\rightarrow C^{12*}$ equilibrium with a small leakage to
stable C^{19}. The reaction rate per alpha particle is

$$\rho_{2\alpha}(\text{He}^6) = \frac{1}{\tau_{2\alpha}(-\text{He}^6)} = \frac{1}{\Gamma_e} \frac{dx_a}{dt}$$

$$= \frac{8\pi^3}{M^2_a(kT)^3} \frac{\Gamma_e^2}{\Gamma_e + \Gamma_a} \times \exp(-Q/kT),$$

where the appropriate $Q$ is $Q(C^{13+} - 3\text{He}^4) = 372 \pm 4$ kev. Numerically:

$$\rho_{2\alpha}(\text{He}^6) = \frac{1}{\tau_{2\alpha}(\text{He}^6)} = 2.37 \times 10^{-4} \frac{\Gamma_e}{T_s^2} \times \exp(-43.2/T_s) \text{ sec}^{-1},$$

where $\Gamma_e$ is in ev and $T_s$ is the temperature in units of $10^8$ degrees. Electron screening will increase this rate by a small factor. Using Salpeter’s expression for weak screening (Sa54), we find that $f_{2\alpha} \sim \exp(0.88(pS/T_s)^{1/2})$, where $p_S$ is the density in $10^6$ g/cm$^3$. Kavanagh (Ka56) finds that $\Gamma_e \gg \Gamma_a$ for C$^{13+}$, so that $\Gamma_a$ does not appreciably influence the reaction rate numerically. Farrell (Fa57), Salpeter (Sa57), and Hayakawa et al. (Ha56b) have calculated $\Gamma_a$ on a reasonable model for states of C$^{12}$ and find $\Gamma_a \sim 0.001$ ev. If the $\gamma$ radiation is highly forbidden then $\Gamma_a$ must be replaced by the width for pair emission by C$^{12+}$. From the inelastic scattering (Fr55) of high-energy electrons by C$^{12}$ this can be calculated (Fr56, Op39) to be $\Gamma_{\text{el}} \sim 5 \times 10^{-4}$ ev.

The present rate is in reasonable agreement with that given by Hoyle (Ho54), although it is worth pointing out that two changes have been made from these earlier calculations. Formerly $\Gamma_a$ was taken as 1 ev, thereby exaggerating the reaction rate. On the other hand, the resonance level was formerly taken such that $Q(C^{13+} - 3\text{He}^4) = 420$ kev. This reduced the reaction rate by an amount that closely compensated for the error in $\Gamma_a$.

The energy generation, with $Q(3\text{He}^4 - C^{19}) = 7.281 \text{ Mev} = 1.165 \times 10^{-8}$ erg, becomes

$$\epsilon_{2\alpha} = 1.39 \times 10^{14} \rho S_a \frac{\Gamma_e}{T_s^2} \exp(-43.2/T_s) \text{ ergs g}^{-1} \text{ sec}^{-1}.$$

If the C$^{12}$ formation is followed by rapid production of O$^{18}$ and Ne$^{20}$, then $Q(5\text{He}^4 - \text{Ne}^{20}) = 19.18 \text{ Mev} = 3.072 \times 10^{-8}$ erg and the numerical coefficient in the above equation becomes $3.66 \times 10^{10}$. Salpeter (Sa57) and Hayakawa et al. (Ha56b) have shown that this rate of energy generation is ample to supply the energy emitted by a star while it is on the tip of the red-giant branch when $\rho \sim 10^5$ g/cm$^3$, $x_\alpha \sim 1$, and $T \sim 10^9$ degrees. Under these conditions $\epsilon_{2\alpha} = 600$ ergs g$^{-1}$ sec$^{-1}$ and $\epsilon_{3\alpha} = 1500$ ergs g$^{-1}$ sec$^{-1}$.

(2) Rate of the C$^{12}(\alpha, \gamma)O^{16}$ Reaction

The C$^{12}$ produced in the helium fusion can capture an alpha particle to form O$^{16}$ and thus continue the element synthesis. The binding energy for an alpha particle in O$^{16}$ is 7.148 Mev, so that this process proceeds mainly through the bound state in O$^{16}$ at 7.116 Mev. This state has spin and parity equal to $1^-$ and thus the alpha capture by C$^{12}(0^+)$ is a $p$-wave capture and the radiation to the ground state of O$^{16}(0^+)$ is electric dipole. The probability for this radiation is somewhat reduced by the isotopic spin selection rule for $T = 0 \to 0$ for electric dipole transitions. Swann and Metzger (Sw56) find the mean lifetime of the state to be $5.0 \pm 0.1 \times 10^{-14}$ sec, or $\Gamma_{\text{el}} = 0.13$ ev. The reduced alpha particle width for the state can be taken as $\theta_{2\alpha} = 0.1$ with a probable variation of a factor of 10 either way (Aj55). The cross-section factor becomes

$$\frac{3.8 \times 10^7}{(E_\alpha + 32)^2} \text{ kev barns}$$

for $E_\alpha = 199 T_s^{1/4}$ kev. The reaction rate per C$^{12}$ nucleus is

$$\frac{1}{\tau_a(C^{12})} = \frac{\rho S_a}{\tau_{12}} \frac{dx_{12}}{dt}$$

$$= 1.44 \times 10^7 \rho S_a T_s^{-1} \exp(-69.19 T_s^{-1}) \text{ sec}^{-1}$$

$$= 1.38 \times 10^{10} \rho S_a \frac{1}{T_s^2 (1+0.16 T_s^{-1})^2} \exp(-69.19 T_s^{-1}) \text{ sec}^{-1}.$$
(3) Further Alpha-Capture Reactions

The rate of the O\(^{16}(\alpha,\gamma)\)Ne\(^{20}\) process depends critically on whether the excited states in Ne\(^{20}\) at 4.95 and 5.62 Mev have the proper spin and parity (even-even or odd-odd) to be formed by O\(^{16}\) and the alpha particles in their ground states. These states probably have even parity and even spin and since they occur at 204 kev and 874 kev, respectively, in O\(^{16}\) and O\(^{14}\) and since \(E_{n} = 246T_{3}^{4}\) kev it is clear that for \(T \geq 10^8\) degrees they will act as resonances in the conversion of O\(^{16}\) into Ne\(^{20}\). Production of Ne\(^{20}\) in comparable or greater amounts than O\(^{16}\) in helium burning is thus to be expected. Resonances can also be expected to occur in Ne\(^{20}\) (\(\alpha,\gamma\))Mg\(^{24}\) but the Mg\(^{24}\) production will be small because of the large barrier factor for alpha particles.

In Fig. III,3 and Table III,3 we show the mean lifetimes versus temperature for the 3He\(^{+}\)C\(^{12}\) and C\(^{12}(\alpha,\gamma)\)O\(^{16}\) reactions as calculated above, as well as the lifetimes for O\(^{16}(\alpha,\gamma)\)Ne\(^{20}\) and Ne\(^{20}(\alpha,\gamma)\)Mg\(^{24}\) as calculated by Salpeter (Sa57). Salpeter uses a dimensionless alpha-particle width, \(\theta_{\alpha} = 0.02\), for O\(^{16}(\alpha,\gamma)\). We have taken \(\theta_{\alpha} = 0.01\) for Ne\(^{20}(\alpha,\gamma)\) and have modified his calculations accordingly. We also show in Fig. III,3 the nonresonant 3He\(^{+}\)C\(^{12}\) lifetime to illustrate the large factor arising from the resonance predicted by Hoyle.

In comparing the rates of the helium-burning reactions with the atomic abundances of C\(^{12}\), O\(^{16}\), Ne\(^{20}\), and Mg\(^{24}\) two points must be borne in mind. In the first place only the rate of the 3He\(^{+}\)C\(^{12}\) is known from experimental evidence to better than a factor of ten. The others are very uncertain and one of the most pressing current problems in element synthesis is the need for experimental measurements on C\(^{12}(\alpha,\gamma)\), O\(^{16}(\alpha,\gamma)\), and Ne\(^{20}(\alpha,\gamma)\). Measurements at high energies (2–5 Mev), where the reactions can be easily detected, are needed to serve as guides in extrapolating to lower stellar energies (0.2–0.5 Mev). Such measurements are planned in the near future at the Kellogg Radiation Laboratory.

In the second place, the relative rates of the reactions depend critically on temperature, and thus different astrophysical circumstances will lead to marked differences in abundance ratios. Helium burning can result, for example, in large or small C\(^{12}/\)Ne\(^{20}\) ratios.

At the same time it is clear from Fig. III,3 that in the region 1.0 to 1.3\(\times\)10\(^8\) degrees the reaction rates are all of the same order of magnitude within the present uncertainties except for the very slow production of Mg\(^{24}\) from Ne\(^{20}\). Thus the atomic abundance ratios of C\(^{12}/\)O\(^{16}\):Ne\(^{20}/\)Mg\(^{24}\) = 1:6:2.0, given by Suess and Urey (Su56), or 1:5:8:1, given by Aller (Al57a) for early-type stars, are reasonable values on the basis of the synthesis of these elements in helium burning. Also other processes are involved. A considerable amount of C\(^{12}\) has been converted into N\(^{14}\) by the CN cycle (see Secs. III F and XI A); Mg\(^{24}\) has been produced in the \(\alpha\) process (see Sec. III D) which consumes O\(^{16}\) and Ne\(^{20}\), and so on.

### D. \(\alpha\) Process

As has just been discussed, helium-burning synthesizes C\(^{12}\), O\(^{16}\), Ne\(^{20}\), and perhaps a little Mg\(^{24}\). This occurs at temperatures between 10\(^8\) and 2\(\times\)10\(^8\) degrees and results in the exhaustion of the helium produced in hydrogen-burning. An inner core of C\(^{12}\), O\(^{16}\), and Ne\(^{20}\) develops in the star and eventually undergoes gravitational contraction and heating just as occurred previously in the case of the helium core. Calculations of stellar evolutionary tracks have not yet been carried to this stage, but it is a reasonable extrapolation of current ideas concerning the cause of evolution into the giant stage. Gravitation is a “built-in” mechanism in stars which leads to the development of high temperature in the ashes of exhausted nuclear fuel. Gravitation takes over whenever nuclear generation stops; it raises the temperature to the point where the ashes of the previous processes begin to burn. Implicit in this argument is the assumption that mixing of core and surrounding zones does not occur.

No important reactions occur among C\(^{12}\), O\(^{16}\), and Ne\(^{20}\) until significantly higher temperatures, of the order of 10\(^8\) degrees, are attained. Two effects then arise. The \(\gamma\) rays present in the thermal assembly become energetic enough to promote Ne\(^{20}(\gamma,\alpha)\)O\(^{16}\). This is the first (\(\gamma,\alpha\)) reaction to occur since the alpha-particle binding in Ne\(^{20}\) is only 4.75 Mev, while it is 7.37 Mev in C\(^{12}\) and 7.15 Mev in O\(^{16}\). The proton and neutron binding energies in these nuclei are 12 to 19 Mev, so that (\(\gamma,\alpha\)) precedes (\(\gamma,p\)) or (\(\gamma,n\)). The alpha

---

**Table III,3. Mean lifetimes in helium burning (years).**

<table>
<thead>
<tr>
<th>(T, \times 10^8) K</th>
<th>(\log_{10} \rho \times 10^{17} ) 3He(^{+})C(^{12})</th>
<th>C(^{12}(\alpha,\gamma))</th>
<th>N(^{14}(\alpha,\gamma))</th>
<th>O(^{16}(\alpha,\gamma))</th>
<th>Ne(^{20}(\alpha,\gamma))</th>
<th>C(^{12}(\alpha,\beta))</th>
<th>Ne(^{20}(\alpha,\beta))</th>
</tr>
</thead>
<tbody>
<tr>
<td>70</td>
<td>15.47</td>
<td>+13.11</td>
<td>+13.6</td>
<td>11.6</td>
<td>17.9</td>
<td>+5.44</td>
<td>+16.3</td>
</tr>
<tr>
<td>100</td>
<td>7.90</td>
<td>7.57</td>
<td>9.60</td>
<td>7.5</td>
<td>12.6</td>
<td>1.93</td>
<td>10.86</td>
</tr>
<tr>
<td>120</td>
<td>5.01</td>
<td>5.94</td>
<td>7.78</td>
<td>5.9</td>
<td>10.1</td>
<td>0.31</td>
<td>8.31</td>
</tr>
<tr>
<td>150</td>
<td>2.18</td>
<td>4.09</td>
<td>5.70</td>
<td>4.4</td>
<td>7.3</td>
<td>-1.54</td>
<td>5.41</td>
</tr>
<tr>
<td>175</td>
<td>0.59</td>
<td>2.90</td>
<td>4.36</td>
<td>3.5</td>
<td>5.5</td>
<td>-2.73</td>
<td>3.39</td>
</tr>
<tr>
<td>200</td>
<td>-0.58</td>
<td>-1.91</td>
<td>3.26</td>
<td>2.9</td>
<td>4.0</td>
<td>-3.71</td>
<td>2.03</td>
</tr>
<tr>
<td>250</td>
<td>-2.16</td>
<td>-0.39</td>
<td>1.54</td>
<td>2.0</td>
<td>1.7</td>
<td>-5.23</td>
<td>-0.38</td>
</tr>
<tr>
<td>300</td>
<td>-3.18</td>
<td>-0.77</td>
<td>0.23</td>
<td>1.5</td>
<td>-0.1</td>
<td>-6.39</td>
<td>-2.20</td>
</tr>
</tbody>
</table>
particles thus released can now penetrate the Coulomb barrier of the remaining Ne\(^{20}\) quite readily at 10\(^9\) degrees, thereby forming Mg\(^{24}\). The resulting transformation is strongly exothermic, viz.:
\[
\text{Ne}^{20} + \gamma \rightarrow \text{O}^{16} + \text{He}^4 - 4.75 \text{ Mev} \\
\text{Ne}^{20} + \text{He}^4 \rightarrow \text{Mg}^{24} + \gamma + 9.31 \text{ Mev}.
\]
Combining these two equations yields:
\[
2\text{Ne}^{20} \rightarrow \text{O}^{16} + \text{Mg}^{24} + 4.56 \text{ Mev}.
\]
Of course, a proportion of the released alpha particles is consumed in scouring out the C\(^{12}\) and forming O\(^{16}\). The result is the formation of additional O\(^{16}\) and Mg\(^{24}\); this is highly favored on a statistical basis even though complete equilibrium is not attained.

The addition of the alpha particles released from the Ne\(^{20}\) need not be limited to the formation of Mg\(^{24}\). Thus, once some Mg\(^{24}\) is produced we also expect Mg\(^{24}\). (\(\alpha, \gamma\))Si\(^{28}\) to take place, since it is also possible to penetrate the Coulomb barrier of Mg\(^{24}\) at temperatures of order 10\(^9\) degrees. Again, once an appreciable concentration of Si\(^{28}\) is built up, we expect Si\(^{28}\).(\(\alpha, \gamma\))Si\(^{28}\) to take place in some degree, and so on for the production of A\(^{28}\) and Ca\(^{40}\).

This is the \(\alpha\) process marked in Figs. 1, 1 and 1, 2, and responsible in our view, for building, in decreasing proportion, the \(\alpha\)-particle nuclei Mg\(^{24}\), Si\(^{28}\), S\(^{32}\), A\(^{38}\), and Ca\(^{40}\). Their relative atomic abundances are indicated in Table III, 4. The abundance of Ne\(^{20}\) is included for comparison. The proportions decrease in general along this sequence, partly because the production of any member of the sequence does not take place in an appreciable amount until the preceding member has first been produced in considerable concentration, and partly because Coulomb effects become increasingly inhibitory as larger and larger values of Z are reached, even at temperatures somewhat above 10\(^9\) degrees.

The nuclei produced in the \(\alpha\) process stand out in abundance above other neighboring nuclei. This is made clear in Fig. VI, I which is discussed in Sec. VI in connection with the neutron-capture process which synthesizes these nuclei. Mg\(^{24}\), Si\(^{28}\), etc., are much too abundant to have been produced in the \(s\) process. The \(\alpha\) process is of course, very similar to helium burning. We differentiate the two processes on the basis that the alpha-particle sources are quite different in the two cases. Detailed calculations on the rates of the \(\alpha\)-process reactions have been given by Hoyle (Ho54), by Nakagawa \textit{et al.} (Na56), and by Hayakawa \textit{et al.} (Ha56b). We estimate that a progression in temperature from 10\(^9\) to 3 \times 10\(^9\) degrees, either in various regions or at various times in a star, will suffice to bring the rates of the \((\alpha, \gamma)\) reactions from Mg\(^{24}\) to Ca\(^{40}\) into reasonable correspondence.

The \(\alpha\) process is, of course, an oversimplification as temperatures near 3 \times 10\(^9\) degrees are considered. Interactions between the heavier nuclei themselves must also be taken into account. It is interesting to note that experimental information on the cross sections of these interactions are already becoming available from research with high-energy accelerators equipped with ion sources capable of producing heavy ions (Ha56a).

One interesting elaboration of the \(\alpha\) process has not been previously discussed in the literature. The straightforward \((\alpha, \gamma)\) reactions involving stable nuclei terminate at Ca\(^{40}\) since Ti\(^{44}\) with an electron capture lifetime of approximately 1000 years (Hu57b) is produced next. We have
\[
20\text{Ca}^{40} + 2\text{He}^4 \rightarrow 22\text{Ti}^{44} + \gamma \\
21\text{Ti}^{44} + e^- \rightarrow 21\text{Sc}^{44} + \nu_e \\
21\text{Sc}^{44} \rightarrow 20\text{Ca}^{44} + \beta^+ + \nu_e \\
\tau_1 \sim 1000 \text{ years} \\
\tau_1 = 3.9 \text{ hours}.
\]

If the alpha-capture lifetimes are greater than 1000 years, then the foregoing decays occur before Ti\(^{44}\) captures another alpha particle. Then one has
\[
20\text{Ca}^{40} + 2\text{He}^4 \rightarrow 22\text{Ti}^{44} + \gamma,
\]
the Ti\(^{44}\) being stable. The important point is that Ti\(^{44}\) is produced in an interaction with approximately the same barrier height as in the case of Ca\(^{44}\) and thus these two should be of roughly equal abundance. This is just the case within a factor of two (favoring Ti\(^{44}\)) in spite of the rapid decrease in abundance evident from Si\(^{28}\) to Ca\(^{44}\). Some Ti\(^{48}\) may also be produced in the \(e\) process discussed in the next section.

A corollary of our argument is that the time-scale of the \(\alpha\) process must exceed 1000 years at Ti\(^{44}\). If Ti\(^{44}\) captured an alpha particle it would produce Cr\(^{48}\) which decays through V\(^{48}\) to Ti\(^{44}\). The unconsumed Ti\(^{44}\) would eventually decay to Ca\(^{44}\), and in this case one would expect Ca\(^{44}\) to be considerably more abundant than Ti\(^{48}\).

E. Succession of Nuclear Fuels in an Evolving Star

Starting with primeval hydrogen condensed into stars, pure hydrogen burning, pure helium burning, and finally the \(\alpha\) process successively take place at the stellar center and then move outward in reaction zones or shells. When the star first contracts the generation of energy by hydrogen burning develops internal pressures which oppose gravitational contraction, and the star is stabilized on the main sequence at the point appropriate to its mass. Similarly, the generation of energy in helium burning should lead to a period of

---

**Table III, 4. Relative abundances of nuclei produced in the \(\alpha\) process (Su56).**

<table>
<thead>
<tr>
<th>(Q_\alpha) (MeV)</th>
<th>Ne(^{20})</th>
<th>Mg(^{24})</th>
<th>Si(^{28})</th>
<th>S(^{32})</th>
<th>A(^{38})</th>
<th>Ca(^{40})</th>
<th>Ti(^{44})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(3.91)</td>
<td>10.00</td>
<td>6.94</td>
<td>6.66</td>
<td>7.04</td>
<td>5.28</td>
<td>9.40</td>
<td>9.32</td>
</tr>
<tr>
<td>Abundance</td>
<td>0.4</td>
<td>0.78</td>
<td>1.00</td>
<td>0.39</td>
<td>0.14</td>
<td>0.052</td>
<td>0.0011</td>
</tr>
</tbody>
</table>

* Produced in helium burning; included for comparison.
SYNTHESIS OF ELEMENTS IN STARS

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relative stability during the red-giant stage of evolution. It is assumed that mixing does not occur. This is substantiated by the fact that, as hydrogen becomes exhausted in the interior, the star evolves off the main sequence which is the location of stars with homogenous interiors.

At the end of helium burning most of the nuclear binding energy has been abstracted, and indeed the cycle of contraction, burning, contraction . . . must eventually end when the available energy is exhausted, that is, when the most stable nuclei at the minimum in the packing fraction curve are reached, near Fe^64. If a star which condensed originally out of pure hydrogen remains stable, it eventually forms the iron-group elements at its center, and this “iron core” continues to grow with time until gravitation, unopposed by further energy generation, leads eventually to a violent instability. This problem of the ultimate instability is discussed, but not solved, in Sec. XII. At this point it suffices only to emphasize that the instability may result in the ejection of at least part of the “iron core” and its thin surrounding shells of lighter elements into the interstellar medium and that in this way a reasonable picture of the production of the abundance peak at the iron-group elements can be formulated. Production of the iron group of elements requires temperatures near 4\times 10^9 degrees, at which statistical equilibrium is reached, as outlined in Sec. II and discussed in detail in Sec. IV (the e process).

F. Burning of Hydrogen and Helium with Mixtures of Other Elements; Stellar Neutron Sources

In the previous discussion we considered the effect of heating hydrogen and its reaction products to very high temperatures. First, the hydrogen is converted to helium, and the resulting helium is converted to C^12, O^16, and Ne^20. Then \( \alpha \) particles released by \( (\gamma, \alpha) \) reactions on the Ne^20 build the \( \alpha \)-particle nuclei Mg^24, Si^28, S^32, A^38, Ca^40, and also Ca^46 and Ti^48. Finally, at very high temperatures, the latter nuclei are converted into the iron group. Further heating of the iron group, although of astrophysical importance, does not lead directly to any further synthesis. Thus all the remaining elements and isotopes must be provided for otherwise than by a cooking of pure hydrogen.

Very much more complicated reactions arise when we consider the cooking of hydrogen and helium mixed with small concentrations of the elements already provided for, e.g., C^12, Ne^20, Fe^64. It is easy to see how such admixtures can arise. Since stars eject the products of nuclear synthesis into the interstellar gas it seems highly probable that only the “first” stars can have consisted of pure hydrogen. The results of hydrogen cooking in such stars would follow the lines described above. But once the interstellar gas was contaminated by this first cooking, nuclear processes would operate on hydrogen which contains impurities. The eventual hydrogen exhaustion will lead to helium burning with impurities. As we shall see later, the presence of other nuclei can lead to highly important effects.

In addition, hydrogen and helium may in some cases become adulterated with impurities even in the “first” stars. For example C^12, Ne^20 built in the inner central regions of such stars may be circulated into the outer hydrogen envelopes. The question whether such mixing occurs, and astrophysical observations bearing on the problem, is considered in Secs. XI and XII.

(I) CN cycle

When C^12 produced in helium burning is mixed with hydrogen at high enough temperatures, hydrogen is converted to He^4 by the CN cycle in addition to the \( pp \) chain previously considered. The implications for energy generation in hot main-sequence stars have been considered by numerous authors since Bethe (Be39) and von Weizsäcker (We38). The reactions of the CN cycle are

\[
\begin{align*}
C^{12}(p, \gamma)N^{13}(\beta^+\nu_e)C^{13} \\
C^{13}(p, \gamma)N^{14} \\
N^{14}(p, \gamma)O^{15}(\beta^+\nu_e)N^{15} \\
N^{15}(p, \alpha)C^{12}.
\end{align*}
\]

These four reactions produce C^13, the heavier stable isotope of carbon, and the two stable forms of nitrogen. For C^13, C^14, and N^14 the \( (p, \alpha) \) reaction is not exothermic and only the \( (p, \gamma) \) reaction occurs. At N^15, the \( (p, \alpha) \) reaction becomes exothermic and much more rapid than the \( (p, \gamma) \) reaction, which serves only as a small leak of material to O^16. The N^15\( (p, \alpha) \) reaction reproduces the original C^12 and a true cycle of reactions is established. Not only does this give rise to the catalytic conversion of hydrogen into helium until hydrogen is exhausted, but it also results in the carbon and nitrogen isotopes not being consumed in hydrogen burning.

The cross sections of the CN-cycle reactions have been determined in the Kellogg Radiation Laboratory for some years and a review of the reaction rates in stars as known up to 1954 is included in the earlier paper by Fowler (Fo54), with numerical computations by Bosman-Crespin et al. (Bo54). New measurements of the CN-cycle reactions in the 100-kev range of interaction energies are now underway by William A. S. Lamb and Ross E. Hester at the Livermore Radiation Laboratory, who are using an ion source capable of delivering several hundred milliamperes of protons in this energy range, a factor of almost 1000 over the currents used in previous experiments. Results on C^12\( (p, \gamma) \) are already published (La57) and results have also been obtained on N^14\( (p, \gamma) \) (La57a). These are the two slowest reactions in the CN cycle and are the ones primarily involved in the over-all rate of this reaction.

In Figs. III,4 and III,5 are shown the yield and
cross-section curves for $^{12}(p,\gamma)$ obtained by Lamb and Hester. For comparison, points obtained originally by Hall and Fowler (Ha50) and by Bailey and Stratton (Ba50) are shown. All of the observations are seen to fit a modification due to Thomas (Th52) of the Breit-Wigner dispersion formula using parameters of the resonance at 456 kev. The new results confirm the original measurements, but more importantly they extend the measurements to cross sections lower by a factor of ten and they do this with increased precision, so that the extrapolation to the lower energies relevant to stellar interiors ($E_{\gamma}\sim 25$ kev) can be made with much more confidence. Their cross section $\sigma = 5.2 \times 10^{-15}$ barn at 90 kev is a representative one and yields

$$S = \left(12 \times 90 \times 5.2 \times 10^{-15}\right) \exp\left(+188.4/(90)^{1/2}\right)$$

$$= 1.7 \text{ kev barns}.$$

Extrapolation to stellar energies yields

$$S_x = 1.2 \pm 0.2 \text{ kev barns for } ^{12}(p,\gamma)N$$

as found by Hall and Fowler (Ha50), so that no change in the numerical calculations of Bosman-Crespin et al. is required.

Experimental results for the reaction $^{14}(p,\gamma)$ have also been obtained and are also shown in Figs. III.4 and III.5. Lamb and Hester find a representative value $\sigma = 7.0 \pm 1.0 \times 10^{-11}$ barn at 125 kev. This corresponds to a cross-section factor

$$S = \left(14 \times 125 \times 7.0 \times 10^{-11}\right) \exp\left(+219.8/(125)^{1/2}\right)$$

$$= 2.8 \text{ kev barns}.$$

Their value, $\sigma = 0.9 \times 10^{-11}$ barn, at 100 kev is the smallest charged particle cross section measured up to the present time.

This and their other values correspond to a cross-section factor at stellar energies equal to $S_x = 3.2$ kev barns. In the Kellogg Radiation Laboratory Fixley (Pi57) has made measurements on this reaction from 220 kev up to 650 kev. His results can be interpreted as indicating that a narrow resonance at 278 kev, with width $\Gamma = 1.7$ kev, is superimposed on a nonresonant background which increases with the energy just as the calculated Coulomb barrier penetration factor times the usual $1/\sqrt{E}$-term. The nonresonant cross section varies from $1.44 \times 10^{-7}$ barn at 450 kev to $4.74 \times 10^{-7}$ barn at 650 kev. The measured cross sections correspond to a cross-section factor at low energies given by $S_x = 2.8$ kev barns. The two sets of experiments indicate that the nonresonant background is characterized by a relatively constant $S$ from 130 to 650 kev. Averaging the two values given above we have, with a reasonable allowance for systematic errors,

$$S_x = 3.0 \pm 0.6 \text{ kev barns for } ^{14}(p,\gamma)O.$$
In spite of the concordance of the new off-resonant measurements for N\(^{14}(p,\gamma)\) there still exists the vexing problem as to whether resonance occurs in N\(^{14}(p,\gamma)\)O\(^{16}\) in the region of stellar energies corresponding to an excitation energy in O\(^{16}\) near 7.37±0.03 Mev. Known levels in O\(^{16}\) near this excitation occur at 6.84, 6.90, 7.61 (\(\frac{1}{2}^+\)), and 8.33 Mev (\(\frac{3}{2}^+\)). The spin and parity assignments for the last two levels come from studies of the N\(^{14}(p,\rho)\) scattering by Pixley (Pi57) and by Hagedorn et al. (Ha57). The 7.61-Mev level is the level investigated in detail at 278-kev bombarding energy by Pixley. It is too narrow and too far removed to influence the cross section at 7.37 kev. In the mirror nucleus N\(^{16}\), which has been investigated more completely, levels occur at 7.16 (\(\frac{1}{2}^+\)), 7.31 (\(\frac{1}{2}^+\)), 7.58 (\(\frac{3}{2}^+\)), and 8.57 Mev (\(\frac{3}{2}^+\)). The spin and parity assignments are the results of a recent comparison of shell model calculations with experiment by Halbert and French (Ha57a). In the light of the spins and parities and of the expected large level shifts (Th52) it can be convincingly argued that the last two levels correspond to the last two indicated above for O\(^{16}\).

One of the other three has not been found in O\(^{16}\). Considerable work has been done on this problem in the last few years, but no new level has been found in this region in O\(^{16}\), and a possible explanation lies in the level shift phenomenon. However, the research has not been exhaustive and it is still possible that a level falls in the range 7.37±0.03 Mev, where it would serve as a resonance at stellar energies and thus would possibly enhance by a considerable amount the N\(^{14}(p,\gamma)\) reaction rate in main-sequence stars.

If resonance in N\(^{14}(p,\gamma)\) occurs then the reaction could be much more rapid than C\(^{12}(p,\gamma)\), in which case the over-all rate of the CN cycle is determined by this last process. In this case the energy generation that is given as \(\epsilon_C\) by Bosman-Crespin et al. (Bo54),

\[\epsilon_C = (3.18±0.54)\times 10^{27}\text{ergs g}^{-1}\text{sec}^{-1},\]

where \(x_C\) is the fraction by mass of C\(^{12}\). This is about 81% of the total carbon, C\(^{12}\) plus C\(^{13}\), and there will be very little nitrogen. Thus \(x_C = 0.8x_{CN}\). The quantity \(\alpha_C\) is given by

\[\alpha_C = \rho C(T_e^{-1} + 0.017) \exp(-136.9T_e^{-1})\]

\[\approx 1.4 \times 10^{-21} \rho C(T_e^{-1} + 0.017) \exp(-136.9T_e^{-1})\]

\[\approx 1.4 \times 10^{-21} \rho C(T_e^{-1} + 0.017) \exp(-136.9T_e^{-1})\]

\[\approx 1.4 \times 10^{-21} \rho C(T_e^{-1} + 0.017) \exp(-136.9T_e^{-1})\]

\[\approx 1.4 \times 10^{-21} \rho C(T_e^{-1} + 0.017) \exp(-136.9T_e^{-1})\]

\[\approx 1.4 \times 10^{-21} \rho C(T_e^{-1} + 0.017) \exp(-136.9T_e^{-1})\]

with the electron shielding factor given by Salpeter's strong screening formula (Sa54) as \(f_C = \exp(0.98\rho/T_e)\).

The mean lifetime of hydrogen becomes

\[\tau_C(H) = 1.90(x_H/\epsilon_C) \times 10^{10}\text{years}\]

\[= 6.06(\alpha_Cx_C)^{-1} \times 10^{-14}\text{years}\]

while for C\(^{13}\) the mean cycle time is

\[\tau_H(C^{13}) = 0.63(x_C/\epsilon_C) \times 10^{10}\text{years}\]

\[= 0.20(\alpha_Cx_H)^{-1} \times 10^{-14}\text{years}\]

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\[= 0.20(\alpha_Cx_H)^{-1} \times 10^{-14}\text{years}\]

On the other hand, if resonance does not occur in N\(^{14}(p,\gamma)\) then we must use the new value for \(S_\alpha\) already given. Ignoring all small correction terms and defining

\[\alpha_N = f_N(T_e^{-1} + 0.017) \exp(-136.9T_e^{-1})\]

\[\approx 1.4 \times 10^{-21} \rho C(T_e^{-1} + 0.017) \exp(-136.9T_e^{-1})\]

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\[\approx 1.4 \times 10^{-21} \rho C(T_e^{-1} + 0.017) \exp(-136.9T_e^{-1})\]

with the electron shielding factor given by Salpeter's strong screening expression as \(f_N = \exp(1.11\rho/T_e)\), then the reaction rate of the CN cycle per gram of material per second becomes

\[r_N = (1.96±0.39)(\alpha_Nx_Nx_{CN}) \times 10^{12}\text{reactions g}^{-1}\text{sec}^{-1},\]

where \(x_N\) is the fraction by mass of N\(^{14}\). In this case almost all of the carbon and nitrogen will be in the form of N\(^{14}\). Thus \(x_N \approx x_{CN}\). The mean reaction rate per proton with four protons consumed per cycle is

\[\phi_N(H) = \frac{1}{x_H} \frac{dx_N}{dt} = (1.31±0.26)(\alpha_Nx_Nx_{CN}) \times 10^{10}\text{sec}^{-1}.\]
The energy release per cycle, excluding the 6% neutrino energy loss, is $25.04\text{ MeV} = 4.011 \times 10^{-5} \text{ erg}$ using Wapstra's masses (Wa55), so that the rate of energy generation can be written

$$e_N = (0.786 \pm 0.16) \langle \alpha_N x_H x_N \rangle \times 10^{38} \text{ ergs g}^{-1} \text{ sec}^{-1}.$$  

The mean lifetime of hydrogen becomes

$$\tau_N (H) = 1.90 \langle x_H / e_N \rangle \times 10^{11} \text{ years}$$  

$$= (4.24 \pm 0.48) \langle \alpha_N x_H \rangle^{-1} \times 10^{17} \text{ years},$$

while for $N^1$ the mean cycle time is

$$\tau_H (N) = 0.55 \langle x_N / e_N \rangle \times 10^{11} \text{ years}$$  

$$= (0.70 \pm 0.14) \langle \alpha_N x_N \rangle^{-1} \times 10^{17} \text{ years}.$$

At the present time the rate of energy generation by the CN cycle can be taken as lying between the wide limits given by $e_N$ and $e_C$. Since resonance has not been established in $N^1(\alpha \gamma)$, the best guess is $e_N$. Values for $e_C$ and $e_N$ and the various lifetimes and cycle times are given in Figs. III,1 and III,2 and in Tables III,1 and III,2.

Because of the uncertainty in the resonance problem in $N^1(\alpha \gamma)$ it is reasonable to employ astrophysical arguments in search of a more definite answer concerning the rate of the CN cycle. Let it be assumed that the observed cosmic abundances of $N^1$ and $C^3$ have been produced at equilibrium at temperature $T$ from $C^2$ in the CN cycle. Then if $S_a = 3.3 \text{ kev barns}$ (Fo54) for $N^1(\alpha \gamma)$ and $S_a = 6.1 \text{ kev barns}$ (Fo54) for $C^3(\alpha \gamma)$ it can be calculated that the relative numbers of nuclei are given by

$$N^1/C^3 = \left[ 133/T_s \right]^{3/2}.$$  

The relative abundance given by Suess and Urey (Su56) is

$$N^1/C^3 = 168.$$  

The temperature obtaining this ratio is approximately $10^7$ degrees. At this temperature the $C^2$ cycle time is $3 \times 10^6$ years for $\rho_{\text{HII}} = 100 \text{ g/cm}^3$, so conversion of $C^3$ to $C^4$ or $N^1$ will occur only in old stars.

With the lower limit of $S_a = 3.0 \text{ kev barns}$ for $N^1(\alpha \gamma)$ it is found that

$$N^1/C^3 = \left[ 440/T_s \right]^{3/2}.$$  

The temperature corresponding to the observed relative abundance is $T = 3.4 \times 10^7$ degrees. This value is close to the temperature believed to hold in the hydrogen-burning shells surrounding the helium cores of red giant stars (Ho55). It is perhaps reasonable astrophysically to argue that this constitutes the last situation in which $C^2$ and $N^1$ are in equilibrium under hydrogen-burning. The $C^2$ and $N^1$ from these shells will be ejected into space when instability arises in the stellar interior. $C^2$ and $N^1$ which remain in the helium region after the exhaustion of hydrogen are consumed in helium burning by $C^3(\alpha n)$ and $N^1(\alpha \gamma)$. On the basis, then, of this argument, the lower limit for $S_a$ would seem to be the preferred value.

With the new value for $S_a$ for $N^1(\alpha \gamma)$ it is found that the CN cycle rate will exceed that for the $pp$ chain only in stars with central temperatures over $18 \times 10^6$ degrees. The existence of nitrogen-rich peculiar stars, as discussed in Sec. XI, is to be expected on this basis since at equilibrium in the CN cycle we now have

$$N^1/C^3 = \left[ 200/T_s \right]^{3/2}$$

so that $N^1 > C^3$ at all reasonable temperatures ($T_s < 40$) for hydrogen burning. Stars in which carbon from the helium-burning cores has been passed slowly enough through hydrogen-burning zones and then mixed into the outer atmosphere will show a much greater over-abundance of nitrogen than of carbon. The $C^1/C^3$ ratio by number should be 4.6 for the carbon in such stars. On the other hand, stars in which $C^3$, $O^4$, and $Ne^5$ from helium-burning zones were mixed quickly without hydrogen-burning into the envelope would show anomalous abundances of these isotopes and not necessarily of $N^1$ or $C^3$. The ratio of $C^1/C^3$ might be quite large in such stars. These points have been emphasized by Greenstein (Gr54a); see also Sec. XI A. Finally, with the new nonresonant value for $N^1(\alpha \gamma)$ the CN cycle is very slow in the sun. The recent inhomogeneous model calculations of Schwarzschild, Howard, and Härm (Sc57) which use only the $pp$ chain need not be modified to include CN cycle processing of hydrogen except for the cases involving rather low values of the initial hydrogen content. D. E. Osterbrock (private communication) has kindly made differential corrections to the calculations of Schwarzschild et al. using the $pp$ reaction and $N^1(\alpha \gamma)$ reaction rates given in this paper. He finds $T_{s} = 15.4 \times 10^6$ degrees and $\rho_{s} = 147 \text{ g/cm}^3$ for an original hydrogen content of 80% by mass. These values replace $T_{s} = 14.8 \times 10^6$ degrees and $\rho_{s} = 132 \text{ g/cm}^3$. For the present values $x_{H} = 0.30$ and $x_{CN} = 0.003$ at the solar center he finds $\epsilon_{pp} = 5.25 \text{ ergs g}^{-1} \text{ sec}^{-1}$ and $\epsilon_{CN} = 0.20 \text{ erg g}^{-1} \text{ sec}^{-1}$. Thus the CN-cycle energy generation is only about 4% of that of the $pp$ chain at the center and is less than this value on the average.

(2) Other Hydrogen-Burning Reactions

The reactions of protons and other light nuclei have been discussed by Fowler (Fo54) and by Salpeter (Sa55) and most recently by Marion and Fowler (Ma57). The rapid destruction of the lithium, beryllium, and boron isotopes in hydrogen burning, the end result being the production of helium, is well known and will not be discussed here, though some of the problems which this raises for synthesis of these elements are discussed in Sec. X.

At the temperatures occurring in the hydrogen-burning shells of red giant stars $O^3$ is converted to
\[ N^{14} \text{ by} \]
\[ O^{17}(p, \gamma) F^{17}(p \pi^+ \nu) O^{18} \]
\[ O^{17}(p, \alpha) N^{14} + 1.20 \text{ MeV (exothermic)}. \]

The N\(^{14}\) is then processed by the CN cycle. When the small leakage due to N\(^{14}(p, \gamma) O^{16}\) from the CN cycle is taken into account with the above reactions one has a generalized CNO cycle.

For O\(^{16}(p, \gamma)\) Tanner and Pixley (Ta57a) give the preliminary value
\[ S_n = 4 \pm 2 \text{ kev barns for } O^{16}(p, \gamma) \]
so that the lifetime for O\(^{16}\) in hydrogen burning becomes
\[ \tau_{H}(O^{16}) = 0.5(\alpha_0 \tau_1)^{-1} \times 10^{-17} \text{ year}, \]
where
\[ \alpha_0 = \rho T_e^{-1} \exp(-166.7T_e^{-1}). \]

Here we have neglected the weak electron shielding expected to hold in high-temperature, low-density shells. Since O\(^{16}(p, \gamma)\) is followed rapidly by O\(^{17}(p, \alpha)\) and by hydrogen processing in the CN cycle, the hydrogen lifetime will be the same as that for O\(^{16}\). Whenever O\(^{16}\) burning is not complete, a small amount of O\(^{17}\) \((\sim 10^{-4} \times O^{16})\) remains in equilibrium with O\(^{16}\). This may be sufficient to account for the small concentration of O\(^{17}=4 \times 10^{-4} \times O^{16}\) observed in the oxygen isotope ratios. Apparently, not all of the O\(^{16}\) and O\(^{17}\) have been destroyed in the hydrogen-burning shells of the giant stars during the relatively short life of this evolutionary stage. On the other hand, O\(^{18}=2 \times 10^{-3} \times O^{16}\) is not produced in sufficient abundance by O\(^{17}(p, \gamma) F^{18}(p \pi^+ \nu) O^{18}\) since the \((p, \alpha)\) reaction is much faster than the \((p, \gamma)\) for O\(^{17}\).

The burning of Ne\(^{20}\) in hydrogen is followed by the NeNa cycle which is somewhat similar to the CN cycle,
\[ \text{Ne}^3(p, \gamma) \text{Na}^6(\pi^+ \nu) \text{Ne}^{21} \]
\[ \text{Ne}^{21}(p, \gamma) \text{Na}^{22}(\pi^+ \nu) \text{Ne}^{22} \]
\[ \text{Ne}^{22}(p, \gamma) \text{Na}^{23} \]
\[ \text{Na}^{23}(p, \alpha) \text{Ne}^{20}. \]

New calculations based on recent experimental information have been made for the rates of the NeNa-cycle reactions by Marion and Fowler (Ma57). It is found that Na\(^{28}(p, \alpha)\) is considerably faster than Na\(^{28}(p, \gamma)\), so that little leakage from the cycle occurs to Mg\(^{24}\). A method of obtaining the reduced nucleon emission widths for the corresponding levels of mirror nuclei is presented and is used in particular in the calculation of the Ne\(^{20}(p, \gamma)\) reaction rate. It is found that a bound level in Na\(^{23}\) at Ne\(^{20}+p-26 \text{ kev} \) markedly enhances the rate of this reaction and that \(S_\gamma \approx 12 \text{ kev barns at the temperatures of hydrogen-burning shells. Results of preliminary measurements by} \]

Substantial quantities of Ne\(^{21}\) are produced at the elevated temperatures occurring just before the hydrogen is exhausted in such shells. This will be true only if the carbon, nitrogen, and oxygen isotopes are relatively rare so that hydrogen is not processed too rapidly before the Ne\(^{20}\) can be burned. The Ne\(^{21}\) thus formed becomes important in the subsequent helium burning when the reaction Ne\(^{21}(n, \alpha)\)Mg\(^{24}\) may serve as a source of neutrons for heavy element synthesis.

The mean lifetimes or cycle times of the light nuclei in hydrogen-burning are summarized in Table III,2 and Fig. III,2.

(3) Helium Burning with Reaction Products; Stellar Neutron Sources

The primary products of pure helium burning are C\(^{12}\), O\(^{14}\), and Ne\(^{20}\). When these are subjected to further hydrogen burning, production of all of the stable isotopes from C\(^{12}\) to Ne\(^{26}\), with the exception of N\(^{15}\), O\(^{18}\), and F\(^{19}\), is accounted for. Production of these three exceptions is treated at the end of this section. In hydrogen burning, cyclic processes are established which transform the hydrogen into helium without consuming the cycling, catalytic nuclei. Exceptions are O\(^{16}\) and O\(^{17}\); these nuclei are consumed in hydrogen burning so it is clear that a proportion of these isotopes originally produced has not been subjected to terminal hydrogen burning.

Exhaustion of the hydrogen by the cyclic processes eventually means that the unconsumed isotopes of carbon, nitrogen, neon, sodium, and perhaps oxygen, will be mixed with helium. Upon contraction the mixture comes to a temperature at which helium begins to interact with these nuclei. Numerous reactions occur but by far the most significant are the exothermic \((a, n)\) reactions which provide a source of neutrons for the s and r processes by which the heavy elements can be synthesized.

The first stellar neutron source was proposed by Greenstein (Gr54) and by Cameron (Ca54, Ca55), namely the exothermic reaction:
\[ C^{16}(a, n)O^{16} + 2.20 \text{ Mev.} \]

To this, Fowler et al. (Fo55) added the exothermic \((a, n)\) reactions involving the \(A = 4n + 1\) nuclei, \(\text{viz.,} \)
\[ O^{17}(a, n)\text{Ne}^{20} + 0.60 \text{ Mev} \]
\[ \text{Ne}^{20}(a, n)\text{Mg}^{24} + 2.58 \text{ Mev} \]
\[ \text{Mg}^{24}(a, n)\text{Si}^{28} + 2.67 \text{ Mev} \]
and the special case,
\[ \text{Mg}^{24}(a, n)\text{Si}^{28} + 0.04 \text{ Mev.} \]

The next \(A = 4n + 1\) reaction, Si\(^{28}(a, n)\)Si\(^{29}\), is endothermic by 1.53 \text{ Mev, and} \( \text{Si}^{29}(a, n)A^{28} \) is endothermic by 2.0 \text{ Mev.} \]

Insofar as neutron sources for the s process in red giants are concerned, we must confine our attention to C\(^{12}\) and Ne\(^{21}\). Very little O\(^{17}\) remains for helium burning.
because it is destroyed by $\alpha^{17}(p,\alpha)$ in hydrogen burning. Mg$^{25}$ and Mg$^{26}$ are not produced in hydrogen burning because the hydrogen is consumed in other processes before Mg$^{24}(p,\gamma)$ becomes operative.

Marion and Fowler (Ma57) have recently discussed the rates of the C$^{14}(\alpha,n)$ and Ne$^{23}(\alpha,n)$ reactions. For the C$^{14}(\alpha,n)$ reaction they find $S_\alpha=2.1\times10^{14}T_\alpha^{-1}$ kev barn so that the C$^{14}$ lifetime is

$$T_{\alpha}(\text{C}^{14})=4.9\times10^{-5}\frac{\text{sec}}{T_\alpha}\exp(323T_\alpha^{-1})\text{ years.}$$

For the Ne$^{23}(\alpha,n)$ reaction they find $S_\alpha=1.6\times10^{13}$ kev barns so that the Ne$^{23}$ lifetime is

$$T_{\alpha}(\text{Ne}^{23})=5.6\times10^{-5}\frac{\text{sec}}{T_\alpha}\exp(468T_\alpha^{-1})\text{ years.}$$

A reaction rate of importance in the following discussion is that of N$^{14}(\alpha,\gamma)$, for which it was found that $S_\alpha=1.2\times10^{13}T_\alpha^{-1}$ kev barn and

$$T_{\alpha}(\text{N}^{14})=8.2\times10^{-5}\frac{\text{sec}}{T_\alpha}\exp(360T_\alpha^{-1})\text{ years.}$$

The above lifetimes are indicated graphically in Fig. III,3 and are tabulated in Table III,3.

Neither C$^{14}(\alpha,n)$ nor Ne$^{23}(\alpha,n)$ are wholly free from objection as stellar neutron sources. Cameron (Ca55) and Fowler et al. (Fo55) have discussed the problems which arise if the C$^{14}(\alpha,n)$O$^{18}$ reaction is taken as the principal source of neutrons for heavy element synthesis in helium burning during the giant stage of a star's evolution. The essential difficulty lies in the fact that only a small amount of C$^{14}$ is produced at equilibrium in the carbon-nitrogen cycle; C$^{14}$/C$^{13}=1/4.6$ by number at equilibrium. As a result too few neutrons are produced when C$^{14}$ begins to interact with the helium. The cosmic abundance ratio C$^{14}$/Fe$^{44}=6.4$ implies that 6.4/4.6=1.4 neutrons will become available per iron nucleus and these will only be sufficient to build nuclei slightly heavier than Fe$^{56}$. Another difficulty is the fact that N$^{14}$ is the most abundant of the isotopes at equilibrium in the CN cycle; N$^{14}$/C$^{14}=168$ even at the high temperatures of 3.4x10$^7$ degrees for a hydrogen-burning shell. The reaction N$^{14}(n,\alpha)$C$^{14}$ consumes a large fraction of the neutrons produced in the C$^{14}(\alpha,n)$O$^{18}$ reaction. These difficulties can be avoided, as emphasized by Cameron, with the C$^{12}$, (\alpha,n)O$^{18}$ reaction as the neutron source, if it is postulated that considerable mixing between core and envelope takes place during the giant stage. In this case hydrogen from the envelope interacts with C$^{12}$ produced by 3He$^0$-C$^{12}$ in the core and constantly replenishes the C$^{13}$ supply but at such a rate that little N$^{14}$ is produced by the C$^{12}(p,\gamma)$N$^{14}$ reaction. In addition if C$^{12}$, produced in the hot center of a core, is mixed into the cooler outer regions of the core where C$^{14}$ is burning, then the C$^{14}$ captures the protons from N$^{14}(n,\alpha)$ and replenishes the C$^{14}$, thus permitting the full quota of neutrons to be made available for heavy element synthesis just beyond Fe$^{56}$.

Fowler et al. proposed the Ne$^{23}(\alpha,n)$Mg$^{24}$ reaction as an alternative source of neutrons which would avoid the foregoing difficulties without calling upon core-envelope mixing with its attendant stability problems. In this proposal it is assumed that Ne$^{25}$, previously produced during helium burning at an earlier stage of galactic evolution, is converted into Ne$^{24}$ in the hydrogen-burning shells at 30-50x10$^4$ degrees surrounding the helium-burning cores of red-giant stars. When the shell hydrogen is converted into what then becomes core helium, the Ne$^{24}$ interacts with helium to produce neutrons. The neutrons are captured by iron-group nuclei, also previously synthesized, to produce heavy elements. The atomic abundance ratio Ne$^{24}$/Fe$^{56}=14$ is taken as indicating that ~14 neutrons will become available per iron nucleus, a sufficient number for considerable heavy element synthesis. This suggestion thus requires that:

(i) The reactions Ne$^{20}(p,\gamma)$Na$^{21}(p,\gamma)$Ne$^{22}$ produce Ne$^{21}$ faster than it is destroyed by the Ne$^{21}(p,\gamma)$Na$^{22}$ reaction.

(ii) The Ne$^{21}$ is produced before the hydrogen mixed with the Ne$^{20}$ is exhausted by the pp chain or the CN-cycle reactions.

(iii) The N$^{14}$ is depleted by the N$^{14}(\alpha,\gamma)$F$^{18}$ reaction before the Ne$^{21}(\alpha,n)$Mg$^{24}$ reaction supplies the neutrons.

(iv) The Ne$^{21}$ is consumed by Ne$^{21}(\alpha,n)$Mg$^{24}$ before helium is depleted in the core.

Because of the uncertainty in the reaction-rate calculations presented here, it is not possible to give unequivocal answers as to whether these conditions are satisfied; however, the following qualified statements can be made. With regard to (i), the results incorporated in Table III,2 and Fig. III,2 indicate that Ne$^{21}$ is indeed produced from Ne$^{20}$ faster than it is destroyed. If the NeNa cycle reaches equilibrium, then about 90% of the Ne$^{20}$ could have been converted into Ne$^{21}$ along with small amounts of Ne$^{22}$ and Na$^{20}$. With regard to (ii), it is clear that the conversion of Ne$^{20}$ into Ne$^{21}$ will not occur before the hydrogen is exhausted by the CN cycle or oxygen reactions unless there is only a small concentration of carbon, nitrogen, and oxygen, perhaps 0.1% of that of hydrogen by weight. This is ~8% the normal abundance but could occur in some stars. With regard to (iii) it may be seen from Table III,3 and Fig. III,3 that N$^{14}$ will be fairly well scoured out at low temperatures before Ne$^{21}$ begins to interact, although there is again considerable uncertainty in this conclusion.

With regard to (iv), Table III,3 and Fig. III,3 show that Ne$^{21}(\alpha,n)$ will compete with other helium reactions only at elevated temperatures near 2x10$^8$
degrees in the core. Even then it is about a hundred times slower than the consumption of He by 3He\( ^{\alpha,\alpha} \rightarrow\) C\( ^{12} \) as long as helium is abundant. However, there is a factor of \( x_a^2 \) in the rate of 3He\( ^{\alpha,\alpha} \rightarrow\) C\( ^{12} \) relative to Ne\( ^{21} \), (\( \alpha,\alpha \)) where \( x_a \) is the concentration by weight of the helium. Thus when \( x_a < 0.1 \), the Ne\( ^{21} \) will be consumed. Indeed, we can expect that as \( x_a \) decreases the core temperature will rise to maintain the energy generation and eventually the last He\( ^{4} \) nuclei will rapidly scorch out the Ne\( ^{21} \) at high temperature. In any case, even if the Ne\( ^{21} \) remains impervious to helium burning it will lose its loosely-bound neutron in the subsequent interactions of the medium-weight nuclei.

With the above considerations in mind a tentative picture of the generation of neutrons in red giant cores is as follows. We assume that the core will spend some 10\( ^7 \) to 10\( ^8 \) years at low temperature (20–40\( \times 10^6 \) degrees) while the star slowly progresses into the giant stage, some 10\( ^6 \) years at 50 to 100\( \times 10^6 \) degrees, and then only a brief period, say 10\( ^5 \) years, at 200\( \times 10^6 \) degrees as it rapidly exhausts helium and burns the middle weight elements. The C\( ^{12} \) lifetime to the \( (\alpha,\alpha) \) reaction is about 10\( ^6 \) years at 75\( \times 10^6 \) degrees, so that C\( ^{12} \) will be consumed at this stage. Thus a few neutrons per Fe\( ^{56} \) become available on a time-scale of about 10\( ^5 \) years. We have seen in Sec. II that a time-scale \( < 10^5 \) years per neutron capture is demanded to produce the early part of the abundance distribution built by the s process; this is discussed further, in Sec. VI.

With the consumption of the C\( ^{12} \) in its central regions, the core now heats to greater temperatures and rapidly processes He\( ^{4} \) by 3He\( ^{\alpha,\alpha} \rightarrow\) C\( ^{12} \), etc. Toward the end of the helium burning, instability may well set in and mixing may occur throughout the core and with the envelope hydrogen. New C\( ^{12} \) is produced, Ne\( ^{21} \) burns, and a great flux of neutrons is produced in the final period of about 10\( ^5 \) years. In Sec.II we showed that a shorter time-scale was necessary to account for the s-process abundance distribution for elements with \( A \geq 100 \), where the atomic abundance curve becomes relatively flat. This is discussed further in Sec. VI, and in Sec. XI in connection with the observed overabundances of heavy elements in stars believed to have recently undergone s-process synthesis in their interiors.

Finally, we turn to neutron sources on a very short time-scale, \( \sim 10 \) seconds, as in supernovae. A mechanism by which neutrons could arise in the hot envelope of a supernova following core collapse was proposed by Burbidge, Hoyle, Burbidge, Christy, and Fowler (Bu56). This mechanism is thought to occur in four stages:

(i) A collapse of the inner regions of a highly evolved star occurs leading to the conversion of gravitational energy into other forms of energy, particularly in the envelope of the star where it is converted into thermal energy. A possible cause of such a collapse is discussed in detail in Sec. XII. The imploding envelope reaches 2\( \times 10^8 \) degrees as a result of the conversion of gravitational energy into thermal energy. The density rises to \( \sim 10^8 \) g/cm\(^2\).

(ii) The onset of reactions of the type C\( ^{12} (p,\gamma) N ^{13} \), O\( ^{14} (p,\gamma) F ^{17} \), Ne\( ^{20} (p,\gamma) N a ^{21} \), Mg\( ^{24} (p,\gamma) A l ^{25} \) take place in the outer parts of the star. These occur very rapidly at 2\( \times 10^8 \) degrees. The mean energy production from these reactions may be taken as approximately 2 Mev per proton. With a composition characteristic of a highly evolved star, consisting of approximately equal abundances (by number) of hydrogen, helium, and light nuclei, the energy released in these reactions is sufficient to raise the temperature of the material to a value \( \sim 10^9 \) degrees as the hydrogen is consumed.

If all of the nuclear energy were converted into particle energy, a temperature of 6\( \times 10^9 \) degrees would be attained. This maximum temperature will not be reached, however, because of the conversion of energy into radiation, turbulence, growing magnetic fields and the general expansion of the eventually exploding envelope. However, 10\( ^8 \) degrees will probably be reached during the initial implosion of the envelope into the region vacated by the collapsing core. A radiation temperature of \( \sim 10^9 \) degrees will result from the hydrogen consumption if \( \rho = 10^8 \) g/cm\(^2\).

(iii) At this high temperature, alpha-particle reactions become important, and the crucial neutron-producing reaction is Ne\( ^{21} (\alpha,\alpha) M g ^{24} \), which follows very rapidly on the 23-second beta decay of Na\( ^{23} \).

The mean reaction time for Ne\( ^{21} \) is of the order of 10\( ^{-6} \) sec if \( T \approx 10^9 \) degrees and \( \rho \approx 10^8 \) g/cm\(^2\). The (\( \alpha,\alpha \)) reactions on C\( ^{12} \) and O\( ^{13} \) are not operative under these conditions since N\( ^{13} \) requires 14 min in the mean to decay to C\( ^{12} \) and F\( ^{17} \) requires 100 sec to decay to O\( ^{17} \).

A series of neutron-producing reactions from combined hydrogen and helium burning on Ne\( ^{20} \) and heavier nuclei are shown in Fig. III.6. All of the beta decays shown in this diagram are reasonably short. In general (\( p,\gamma \)) reactions compete with positron emission until proton addition is no longer possible and positron emission must occur.

(iv) The neutrons produced in hydrogen and helium burning are rapidly thermalized (\( kT \approx 100 \) kev) and then they are captured primarily by the abundant nuclei of the iron group which were initially present in the envelope. We have supposed that their abundances at this stage total about 1% by mass. If the envelope was very underabundant in these elements the neutrons would be captured by the light elements. This rapid neutron capture process, the r process, will be discussed in detail in Secs. VII and VIII. For this discussion we require to know the neutron density. This will now be estimated.

The neutrons become available in the mean lifetime of Na\( ^{21} \) (33 sec), or Mg\( ^{24} \) (7 sec) and Al\( ^{26} \) (10 sec). Thus a reasonable mean time for the neutron production during the imploding high-density stage is about 10
sec. There will be one neutron produced for each Ne\textsuperscript{39} nucleus present, and on the assumption that there are approximately equal number densities of hydrogen, helium, and light nuclei, and that perhaps one-third of the light nuclei are together C\textsuperscript{12}, O\textsuperscript{16}, and Ne\textsuperscript{39}, each having equal density, the number density of Ne\textsuperscript{39} is \( \sim 10^{33} \) g/cm\( ^3 \). It has been earlier stated that a reasonable value for \( \rho \sim 10^8 \) g/cm\( ^3 \), during the implosive stage. However, the neutron captures will occur during the later expanding stage when \( \rho \sim 100 \) g/cm\( ^3 \), so that the neutron density during this stage will be \( \sim 10^{34} \) neutrons/cm\( ^3 \). We assume that the radiation temperature remains at the high value of \( \sim 10^8 \) degrees during the capture process. As we shall see in Sec. VII, this means that captures occur until the neutron binding energy is reduced to \( \sim 2 \) Mev. Further captures must wait until beta emission occurs with lifetimes for the neutron-rich nuclei of about 3 sec. Each beta decay is followed by about 3 captures so that the total time for \( \sim 200 \) captures (Fe\textsuperscript{56} \( \rightarrow \) Co\textsuperscript{58}) is \( \sim 200 \) sec. Thus the capture time is long as compared with the total neutron production time and we are justified in assuming that the initial neutron density is that given by the original Ne\textsuperscript{39} density.

To conclude, in our discussion of the \( r \) process in supernova envelopes we put the neutron density \( n_n \sim 10^{34} \) cm\( ^{-3} \) and the temperature \( T \sim 10^8 \) degrees. The thermal neutron velocity at \( kT \sim 100 \) kev is approximately \( 4 \times 10^8 \) cm/sec, so that the neutron flux \( \langle \Phi_n \rangle \sim 4 \times 10^{31} \) neutrons/cm\( ^2 \) sec.

In Secs. X and XII brief mention is made of a further possible neutron source in a very highly collapsed stellar core.

(4) Origin of N\textsuperscript{16}, O\textsuperscript{16}, and F\textsuperscript{19}

The concentration of N\textsuperscript{16} given by the CN cycle is far too low, as compared with that of N\textsuperscript{4}, to explain the ratio N\textsuperscript{16}/N\textsuperscript{4}=1/270 in the atomic abundance distribution. It is too low by a factor \( \sim 100 \), assuming the very slow rate for N\textsuperscript{16}(\( \phi, \gamma \))O\textsuperscript{16}, discussed above in Sec. III F (1). Hence it appears that N\textsuperscript{16} must be produced by reactions other than those of the normal CN cycle. Let us consider the reaction C\textsuperscript{13}(\( \alpha, n \))O\textsuperscript{14}, occurring in the presence of N\textsuperscript{14}, derived previously from the CN cycle, and also in the presence of a strong excess of C\textsuperscript{13} derived from helium burning. This requires mixing of material in a helium-burning core. Protons liberated by N\textsuperscript{14}(\( n, p \))C\textsuperscript{14} are, in the main, converted back to neutrons by the reactions C\textsuperscript{12}(\( \phi, \gamma \))N\textsuperscript{15}(\( \beta, \nu \))C\textsuperscript{13} and C\textsuperscript{12}(\( \alpha, n \))O\textsuperscript{14}, since the C\textsuperscript{13}, in excess over its CN cycle abundance, will capture most of the protons. Nor are neutrons lost by the reaction C\textsuperscript{12}(\( n, \gamma \))C\textsuperscript{13}, again because of the occurrence of the reaction C\textsuperscript{12}(\( \alpha, n \))O\textsuperscript{14}. Thus the neutrons tend to cycle,
either through $N^{14}(\alpha,p)C^{14}$, or through $C^{14}(n,\gamma)C^{14}$, the latter reaction being important because of the excess of $C^{14}$.

Neutrons are eventually lost, however, through capture in reactions with smaller probability. Important among these is $N^{14}(\alpha,\gamma)N^{15}$. If the $C^{14}$ excess is great enough for all protons, released by $N^{14}(\alpha,\gamma)N^{15}$, to be absorbed in $C^{14}(p,\gamma)N^{18}$, the ultimate result is to produce an $N^{18}$ concentration comparable with that of the original $C^{14}$ concentration. Since $C^{14}/N^{14}=1/168$, this implies the possibility of $N^{18}/N^{14} \approx 1/168$. As noted above, the isotopic ratio is $N^{14}/N^{18}=1/270$. It appears, therefore, that the present process, dependent on an excess of $C^{14}$ becoming mixed with the products of the CN cycle in helium burning, is capable of providing for the origin of $N^{16}$.

At somewhat higher temperatures than are required to promote the above reactions, $F^{19}$ is then produced by $N^{14}(\alpha,\gamma)F^{19}$. At the same time $O^{18}$ is produced by $N^{14}(\alpha,\gamma)F^{19}(\beta^{+},\nu)O^{18}$. It would then follow that $F^{19}/O^{18} \approx N^{18}/N^{14}$. This is the case within a factor of 10.

To conclude, detailed mechanisms have been advanced for the production of the nuclei $C^{14}$, $C^{13}$, $N^{14}$, $N^{15}$, $O^{16}$, $O^{17}$, $O^{18}$, $Fe^{19}$, $Ne^{20}$, $Ne^{21}$, $Ne^{22}$, $Na^{19}$ in combination hydrogen-helium-burning in stars. These mechanisms are illustrated graphically in Fig. I.2.

**IV. e PROCESS**

At temperatures above $\sim 3 \times 10^{9}$ degrees all manner of nuclear processes occur in great profusion, i.e., $(\gamma,\alpha)$, $(\gamma,\beta)$, $(\beta,\gamma)$, $(\alpha,\gamma)$, $(\gamma,\nu)$, $(\beta,\nu)$ reactions as well as others involving heavier nuclei. Such reactions allow the conversion of a nucleus $A$, $Z$ to a nucleus $A'$, $Z'$, even in cases in which $Z$ and $Z'$ are large; i.e., even though considerable Coulomb barriers may be involved in the conversion. This is the physical condition for statistical balancing to occur among the nuclei. In previous papers (Hoyle66, Hoyle54), it was shown that the abundances of the elements in the iron peak could be synthesized under conditions of temperature and density such that statistical equilibrium between the nuclei and the free protons and neutrons was achieved. In this section this question is re-examined using the more accurate nuclear and abundance data now available, and taking into account the appropriate corrected nuclear states, the energies and spins for which are available now from the work of Way, King, McGiniss, and Lieshout (Wa55a).

Under conditions of statistical equilibrium the number density $n(A,Z)$ of the nucleus $A$, $Z$ is given by

$$n(A,Z) = \frac{A}{2\pi \hbar^2} \left[ \frac{1}{n_{\nu}} \right]^{A-Z} \frac{Z}{n_{\nu}} \frac{Z}{M} \exp \left[ \frac{Q(A,Z)}{kT} \right],$$

where $n_{\nu}$, $n_{p}$ are the number densities of free neutrons and protons, $M$ is the atomic mass unit, and $Q(A,Z)$ is the statistical weight factor given by

$$Q(A,Z) = \sum (2I_{r}+1) \exp (-E_{r}/kT),$$

where $E_{r}$ is the energy of the excited state measured above the ground level, and $I_{r}$ is the spin. $Q(A,Z)$ is the binding of the ground level of the nucleus $A$, $Z$, and is given by

$$Q(A,Z) = e^{2}[(A-Z)M_{n}+ZM_{p}-M(A,Z)].$$

This is the form given by Hoyle (Ho48) except that formerly the approximation was made of putting $Q(A,Z) = 2$, and a slight numerical change of 0.01 in the numerical term in the right-hand side of the equation has been made. Logarithms are to the base 10.

The nuclear data required to calculate $Q(A,Z)$, $\omega(A,Z)$ are available, so that explicit values of $n(A,Z)$ for various $A$, $Z$ can be worked out if $n_{\nu}$, $\theta$, $T_{b}$ are specified. Thus it might seem, at first sight, as if abundances of the nuclei under statistical equilibrium are functions of three independent parameters. This is not the case, however. Two parameters are sufficient to determine the system. This can be understood best from taking $\rho$, the density, and $T_{b}$ as the independent parameters, although we shall see later that $\theta$, $T_{b}$ give a more convenient choice of parameters from an arithmetical point of view. If $\rho$ is given, the abundances calculated from the above equation must satisfy the condition

$$\sum M_{(A,Z)} n_{(A,Z)} + \sum M_{p} n_{p} + \sum M_{n} n_{n} = \rho,$$

the contribution of the electrons present in the system being neglected. The contribution of the free protons and free neutrons is also negligible for the values of $\rho$, $T_{b}$ used below, although values of $\rho$, $T_{b}$ could be chosen for which this would not be so. Equation (3) can be interpreted in a number of ways, for instance as determining $\log n_{\nu}$ in terms of $\rho$, $T_{b}$, $\theta$. A second condition determining $\theta$ in terms of $\rho$, $T_{b}$ arises in one or other of two alternative ways. If the time-scale is long enough, equilibrium between protons and neutrons is established through the operation of beta processes among the nuclei. This equilibrium is expressed by the equations

$$n_{\nu} = 10^{-\frac{9}{2}} \frac{\rho A \pi^{2} \hbar^{2}}{kT} \exp (Q_{\nu}/kT),$$

$$n_{p} = \sum_{A,Z} Z n(A,Z) + n_{p},$$

$$Q_{\nu} = e^{2}(M_{n}+M_{p}-M_{n}) = -0.78 \text{ Mev},$$

or

$$n_{n} = 10^{-\frac{9}{2}} \frac{\rho A \pi^{2} \hbar^{2}}{kT} \exp (Q_{n}/kT),$$

$$n_{p} = \sum_{A,Z} Z n(A,Z) + n_{p},$$

$$Q_{n} = e^{2}(M_{n}+M_{p}-M_{n}) = -0.78 \text{ Mev},$$

or

$$n_{n} = 10^{-\frac{9}{2}} \frac{\rho A \pi^{2} \hbar^{2}}{kT} \exp (Q_{n}/kT),$$

$$n_{p} = \sum_{A,Z} Z n(A,Z) + n_{p},$$

$$Q_{n} = e^{2}(M_{n}+M_{p}-M_{n}) = -0.78 \text{ Mev}.$$
\( m_e \), being the electron mass, and \( n_e \) the electron number density.

If, on the other hand, the time-scale is not long enough to establish equilibrium between neutrons and protons, then (4) must be replaced by the condition that the ratio of the number of neutrons per unit volume (bound and free together), to the number of protons per unit volume must be a specified constant with a value determined by the initial state of the material. That is to say

\[
\sum_{Z,A} (A-Z)n(A,Z) + n_p = (\text{specified constant}) \\
\times [\sum_{Z,A} Zn(A,Z) + n_p].
\] (5)

Before proceeding to a numerical discussion of the above scheme of equations it is desirable to relate them very briefly to astrophysical considerations, particularly to the temperature evolution and time-scales shown schematically in Fig. II,4. So long as the time-scale is longer than a few months (4) are applicable, but in the final rise of temperature to the explosion point shown in Fig. II,4, and during the cooling that follows explosion, the time-scale is far too short to admit of the use of (4). During these phases (5) must be employed. An examination of (5) has shown that it requires \( \theta \) to stay nearly constant at a value determined by the constant that appears on the right-hand side of the equation. This constant is in turn determined by the values of \( n(A,Z) \), \( n_p \), \( n_n \) that are operative at the last moment of applicability of (4).

We can now give a description of the physical and mathematical conditions that determine the abundances of the nuclei of the iron peak. Astrophysical reasons, discussed in Sec. XII, require a temperature evolution of the form shown in Fig. II,4 to take place in certain types of star. As previously pointed out, when \( T \) rises above about \( 3 \times 10^9 \) degrees the statistical equations become applicable. So long as the time-scale is long enough, beta processes establish the equilibrium that satisfies (4). The decay of Mn\(^{56}\) with a half-life of 2.58 hours is important in this connection, as also is Co\(^{57}\) with a half-life of 267 days. The Co\(^{57}\) turns out to be a fairly abundant nucleus and its decay has the effect of changing protons to neutrons. The Mn\(^{56}\), on the other hand, changes neutrons to protons. The time-scale for an appreciable interchange between neutrons and protons, or vice versa, turns out to be of the order of months. This time is determined by two considerations, the half-lives of the beta-decaying nuclei and their abundances. In the case of Co\(^{57}\) the lifetime is long but the abundance fairly high, as has been mentioned. In the case of Mn\(^{56}\) the lifetime is short but the abundance comparatively low. For the values of \( \rho \), \( T \) of interest in the present section the free neutron density \( n_n \) is too low for free neutron decay to be of much importance in establishing (4).

Reference to Fig. II,4 shows that the time-scale becomes too short to maintain (4) as the explosion point is approached. Equation (5) must then be used, the constant on the right-hand side being obtained by the values of \( n(A,Z) \), \( n_p \), \( n_n \) that are operative in (4) at the time that we switch from (4) to (5). This statement implies a discontinuous transition between (4) and (5). A rigorous treatment would provide for a continuous transition so that in this sense the present arguments are approximate. Throughout the remaining evolution to the explosion point, and in the subsequent cooling, \( \theta \) remains approximately constant. The basic reason for this is that although complete equilibrium between prompt nuclear reactions is obtained the neutron-proton ratio is determined by the beta-decay rates and the majority of these decays no longer have time to take place. The question now arises as to what this constant value of \( \theta \) should be. The values of \( n(A,Z) \), \( n_p \), \( n_n \) at the time (4) ceases to be operative would be sufficient to determine \( \theta \), in addition to determining the constant on the right-hand side of (5). The values of \( n(A,Z) \), \( n_p \), \( n_n \) in question are in turn determined by the values of \( \rho \), \( T_s \) that are operative at the stage at which (4) ceases to hold good. Thus if these values of \( \rho \), \( T_s \) were known, \( \theta \) could be calculated. Unfortunately the relevant values of \( \rho \) and \( T_s \) are not known a priori. Only accurate quantitative calculations of stellar evolution can provide this information, and such calculations have not yet been made.

Our method of solution has been to carry out a series of abundance calculations for a network of values of \( \theta \) and \( T_s \) and to obtain the values of \( \theta \) and \( T_s \) which give the best fit to the observed relative abundances. To do this it is convenient to determine the abundances of all of the isotopes relative to Fe\(^{56}\), which is shown later to be the most abundant nucleus. The equation for Fe\(^{56}\) is obtained by substituting \( A = 56, Z = 26 \) in (2); equations for the abundances of the other isotopes relative to Fe\(^{56}\) are then obtained by eliminating \( n_n \) between the equation for Fe\(^{56}\) and the general equation (2). We then obtain

\[
\log \frac{n(A,Z)}{n(56,26)} = \frac{A - 56}{56} \left[ \log \frac{n(56,26)}{\omega(56,26)} - 33.77 \right] \\
- \frac{3}{2} \log T_s - \frac{3}{2} \log 56 \\
+ \frac{5.04}{T_s} \left\{ \frac{\log (A,Z) - A}{56} \right\} \\
+ \log \left( \frac{\omega(A,Z)}{\omega(56,26)} + \theta \left( \frac{Z - 26}{56} \right)^{2/3} \right) \\
+ \log \frac{\omega(A,Z)}{\omega(56,26)} + \theta \left( \frac{26}{56} \right)^{3/2} \left( \frac{A}{56} \right)^{1/2} \\
+ \log \frac{\omega(A,Z)}{\omega(56,26)} + \theta \left( \frac{26}{56} \right)^{3/2} \left( \frac{A}{56} \right)^{1/2} \\
+ \log \frac{\omega(A,Z)}{\omega(56,26)} + \theta \left( \frac{26}{56} \right)^{3/2} \left( \frac{A}{56} \right)^{1/2} \\
\]

(6)
A considerable simplification can now be effected at the expense of only a slight approximation. This rests on two points, first that \( A = 56, Z = 26 \) is the most abundant nucleus, and second that the application of (6) is confined to values of \( A \) not very different from 56, since nuclei with appreciably different \( A \) turn out to possess only negligible abundances; the exception to this is obtained when \( A = 4, Z = 2 \), so that the present approximation cannot be made if it is desired to work out the \( \text{He}^4 \) abundance. In this approximation \( A/56 \) is close to unity and the last term on the right-hand side of (6) may be neglected. More important, the quantity \((A - 56)/56\) is small compared to unity, a circumstance that enables us to approximate to the factor that multiplies \((A - 56)/56\). For values of \( \rho \) between 10\(^4\) g/cm\(^3\) and 10\(^6\) g/cm\(^3\), which covers the range of density of present interest, we write

\[
A - 56 \approx \frac{n(56,26)}{56} \left[ \log \frac{\omega(56,26)}{56} - 33.77 - \frac{3}{2} \log T_s - \frac{3}{2} \log 56 \right] 
\]

\[
\simeq 0.183(56 - A). 
\]

The coefficient 0.183 is strictly correct for \( T_s = 3, \rho = 2 \times 10^6 \) g/cm\(^3\), but a change to \( \rho = 10^8 \) g/cm\(^3\) only alters the coefficient to 0.153. This approximate term is in any case of comparatively minor importance in its effect on the value of \( \log n(A,Z) \). We now have

\[
\log n(A,Z) = \log(1.83(56 - A)) + \frac{5.04}{T_s} \left[ Q(A,Z) - \frac{Q(56,26)}{56} \right] + \frac{\omega(A,Z)}{\omega(56,26)} + \theta(Z - 0.4644). 
\]

Reference to Fig. II,4 shows that the temperature increases up to the explosion point and cools thereafter. Through the increase of temperature the equilibrium abundances will change in accordance with (7). The abundances will also change during the early part of the cooling. But eventually the temperature will fall so low that the nuclear reactions slow up and the isotopes become "frozen." Freezing occurs when \((\gamma,\alpha)\), \((\gamma,n)\), and \((\gamma,p)\) reactions become insufficient to provide a ready supply of alpha particles and free neutrons and protons. Because the thresholds for these reactions differ from one nucleus to another, freezing will not occur simultaneously for all nuclei, and so it will occur over a range of temperature. Thus the particular value of the temperature that we find to give the best fit to the observed abundances should be interpreted as the mean freezing temperature.

Calculations of relative abundances have been carried out for values of \( T \) ranging from 2.52\times10^9 to 7.56\times10^9 degrees, and values of \( \theta \) of 1.5, 2.5, and 3.5. The binding energies \( Q(A,Z) \) have been obtained from the tables of Wapstra and Huizenga (Wa55a), and the weight factors \( \omega(A,Z) \) have been estimated from the level diagrams of Way et al. (Wa55a). In one case, that of Ni\(^{58}\), an additional binding of 0.84 Mev over that given by Wapstra and Huizenga has been used to bring the \( Q \) into agreement with the mass-spectrometer value.

The best fit is obtained for \( T = 3.78 \times 10^9 \) degrees and \( \theta = 2.5 \). This value of \( \theta \) corresponds to a density of the order of 10\(^4\) g/cc. Results of this calculation are shown in Table IV,1 and in Fig. IV,1. To determine the final abundances of the stable isotopes the last reactions, i.e., the freezing reactions, have to be taken into account, together with the beta processes which take place after the freezing reactions have occurred. These reactions are given in column 4 of Table IV,1. The particular freezing reactions which we have taken into account are \( \text{V}^{47}(\gamma,\alpha) \), \( \text{Mn}^{54}(\gamma,\alpha) \), \( \text{Co}^{60}(\gamma,\alpha) \), and \( \text{Cu}^{64}(\gamma,\alpha) \), all of which have fairly low thresholds of 6–7 Mev and which will freeze at a lower temperature than \((\gamma,\alpha)\) and \((\gamma,n)\) reactions. We have arbitrarily supposed that in these cases half of each of these isotopes have gone to \( \text{Tl}^{205}, \text{C}^{42}, \text{Fe}^{56}, \) and \( \text{Ni}^{62} \), respectively. In columns 6 and 7 of the table and in Fig. IV,1, these calculated abundances relative to \( \text{Fe}^{56} \) have been compared with the solar element abundances given by Goldberg et al. (Go57) taken together with the well-known isotopic abundances. For \( 50 \leq A \leq 62 \) the fit is very good indeed, the standard deviation between calculated and observed values being 0.3 in the logarithm in this region. The binding energies are in general uncertain to within 0.2 Mev, and this uncertainty is sufficient to introduce deviations of the same order as those found.
A similar uncertainty is also present because of our assumption of a unique freezing temperature.

For \( A < 50 \) and \( A > 62 \) there are gross discrepancies between observed and calculated values and because of these we conclude that other modes of synthesis are demanded for these isotopes. Beyond \( A = 62 \), the nuclei Ni\(^{44}\), Cu\(^{63}\), and Cu\(^{64}\) have all been assigned in the appendix to the \( s \) process and in Sec. VI and in Fig. VI,3, it will be seen that the \((\sigma N)\) products for Cu\(^{64}\), Ni\(^{56}\), Zn\(^{66}\), and Cu\(^{64}\) are all comparable \((\sigma N \sim 10^6)\) as is expected in the first part of the \( s \)-process chain which runs from 63 \( \leq A \leq 209 \). The \( \sigma N \) product would not be expected to be constant if these nuclei had been synthesized by the equilibrium process. Below \( A = 50 \), the titanium isotopes have been assigned as follows: Ti\(^{48}\) to the \( s \) process, Ti\(^{44}\) to the \( \alpha \) process, and Ti\(^{47}\), Ti\(^{49}\), Ti\(^{50}\) probably to the \( r \) process. In this region of \( A \), \( 46 \leq A \leq 50 \), where the \( s \), \( \alpha \), \( \epsilon \), and \( r \) processes all intersect, details of the exact contributions from each process are not easy to determine. In particular, the \( r \)-process calculations in this region are very approximate (cf. Sec. VII).

**V. \( s \) AND \( r \) PROCESSES: GENERAL CONSIDERATIONS**

The eventual aim is to compare calculated abundances with observed abundances both for the \( s \) and the \( r \) processes. Our method of assignment to these processes is described in Sec. II. Before we describe in detail the ways in which the two processes can be handled, it is necessary to digress in order (i) to review briefly the reasons from the standpoint of nuclear stability why all three processes, the \( s \), the \( r \), and the \( p \), are demanded, and (ii) to describe our neutron capture cross sections, \( \sigma \), which are given in the appendix and are demanded in the discussion of the \( s \) process alone.

### A. “Shielded” and “Shielding” Isobars and the \( s, r, p \) Processes

Let us consider nuclei of the same \( A \) but different \( Z \). In Fig. V,1 we show the well-known schematic plot of the stability of such nuclei as a function of \( Z \). In this diagram, \( Z_A \) is the charge of the most stable isobar at a given \( A \) as given by the smooth Weizsäcker mass law (We35). It can occur near an odd or even integral value of \( Z \). The coefficient in the quadratic dependence of the mass \( M(A,Z) \) on \((Z-Z_A)^2\) is \( B_A/2 \). For odd \( A \) (top diagram) there is only one parabola and the nucleus with charge \( Z \) closest to \( Z_A \) is the stable isobar. Those with smaller \( Z \) are electron-antineutrino emitters, while those with greater \( Z \) capture electrons and emit neutrinos or emit a positron with a neutrino. In a few cases two nuclei near the bottom of the parabola will apparently be stable, one with a very long but detectable half-life or with too long a half-life to be measured. Rh\(^{87}\) is an example of this type; it decays to Sr\(^{87}\) with a half-life of \( 4.3 \times 10^6 \) years. An odd-\( A \) nucleus produced in neutron capture will always decay eventually to the most stable isobar.

For even \( A \) there are two mass parabolas. Those nuclei that have odd \( Z \), odd \( N \) (\( = A-Z \)) lie on a parabola separated from that for nuclei with even \( Z \), even \( N \) by the pairing energy, \( \delta_A \). As shown in the two bottom
diagrams, one, two, or three nuclei can be stable, depending on the ratio of $\delta_A$ to $B_A$. Double beta decay is possible for all but the most stable, but this is a very slow process with half-life $> 10^{18}$ years. An example of cases where there are three stable isobars is given by Sn$^{134}$, Te$^{134}$, and Xe$^{134}$. On the other hand, there is only one stable nucleus for each even $A$ for $A \leq 34$ and two stable nuclei for $36 \leq A \leq 76$. For $A \geq 78$ there may be two or three stable nuclei for each value of $A$. In a neutron-capture process at a rate rapid compared to negative beta decay (the $s$ process), the nuclei produced lie far up the left-hand side of the parabolas shown. At the termination of the process $\beta^-$ decay leads to the stable isobar with smallest $Z$. This is the *shielding* isobar, and the other one or two stable isobars with the same $A$ are said to be *shielded*. In a neutron-capture process at a rate slow compared to negative beta decay (the $s$ process), the isobar on the left is sometimes produced, that in the middle is sometimes produced, but never the isobar on the right. Those not produced are said to be *by-passed* in the $s$ process. It is to produce the isobar on the right, that having the greatest value of $Z$, that a third process, the $\rho$ process involving the addition of protons on already existing heavy nuclei, is demanded. Details of this process are described in Sec. IX.

**B. Neutron-Capture Cross Sections**

In the third column of the appendix we give either the neutron-capture cross sections or, in the case of unstable nuclei, the half-lives against beta decay. The cross sections ($\sigma$) which are given in millibarns ($10^{-37}$ cm$^2$) are mostly cross sections for neutron capture with emission of gamma radiation $(n,\gamma)$. For small $A$ a few cross sections for other reactions initiated by neutrons, in which there is no increase in $A$ \{[$(n,p)$ reactions] or a decrease in $A$ \{[$(n,\alpha)$ reactions], are given. All of the values are for a neutron energy of 15 keV which is reasonable if the $s$ process is taking place in the interiors of red-giant stars at $T=1-2 \times 10^8$ degrees. To obtain the cross sections several sources were used:

(i) Many of the $\sigma$'s for the lighter nuclei have been obtained from the Brookhaven tables (Hu55a). Others have been obtained from the thermal cross sections (1/40 ev) by extrapolation, using a $1/\nu$ law, where this law is thought to apply (designated 0.1% of thermal in the appendix). Where $\sigma$ does not drop as rapidly as $1/\nu$ in the kilovolt region we have taken 1% of thermal (designated this way in the appendix) to be a reasonably accurate value. In a few cases the $\sigma$'s have been marked extr. meaning extrapolated from nearby experimental values, or exp. meaning experimentally measured.

(ii) The majority of the remaining $\sigma$'s have been obtained from the pile reactivity measurements made by Snyder et al. (Sn55), in which reactivity changes due to different materials are measured. Since the reactivity measure is given in units of cents per mole, normalization has been carried out by using the $\text{Bi}^{209}(n,\alpha)$ reaction which has a cross section of 1 barn at 15 keV. No reactivity measures have been given for isotopes with $Z<17$, and so we have made a reasonable extrapolation of the reactivity curves of Snyder et al. using the independently measured cross section for $\text{Al}^{27}$ of 5.2 mb. The normalization by means of the $\text{Bi}^{209}(n,\alpha)$ reaction for isotopes with $Z>17$ has been carried out by putting
1 barn equal to 24 cents per mole for nonmagic isotopes with odd \( A \), the original normalization of Snyder et al., and 1 barn equal to 8 cents per mole for the even \( A \) and the nuclei with closed neutron shells. This latter re-normalization has been carried out after comparison of the cross sections obtained from the reactivity measurements with experimental activation measurements for a neutron energy of “25 kev” made at Oak Ridge (La57b) and at Livermore (Bo57). These latter cross sections are shown in Table VI, and they are discussed further in Sec. VI when the \( \sigma N \) products are considered. In the appendix the cross sections of both even and odd isotopes with closed neutron shells were increased over the original estimates, using the original normalization of Snyder et al., in order to avoid making the even \( A \) cross sections exceed the odd \( A \) cross sections at these magic-neutron numbers. A further effect which must arise and which cannot presently be estimated from the reactivity measures is the variation in \( \sigma \) within isotopes of even \( A \). This variation must be in the sense that the lightest isotopes will tend to have larger cross sections than the heavier ones. Trends such as these, which may affect the run of the \( \sigma N \) products, are discussed further in Sec. VI.

The reason for differentiating between \( \sigma \)'s for odd and even \( A \) instead of differentiating between nuclei with odd and even numbers of neutrons is the following. The results of Hurwitz and Bethe (Hu51) show that neutron capture occurs through a compound nuclear state whose effective excitation should be measured without regard for special pairing effects in the ground state of this nucleus. Thus only odd-even effects in the ground state of the target nucleus are relevant. The even \( A \)-even \( Z \)-even \( N \) target nuclei capture neutrons with less excitation and thus smaller \( \sigma \)'s than the odd \( A \) target nuclei. In particular, the value of \( \sigma \) for odd \( A \) nuclei is the same for the odd \( Z \)-even \( N \) case as for the even \( Z \)-odd \( N \) case. Snyder et al., have taken this effect into account, and in fact their plotted curves give a large odd-even effect. The difference between the normalizations that we have used for odd and even \( A \) is in order to diminish the odd-even effect.

**Fig. V.2.** The neutron capture paths of the \( s \) process and the \( r \) process. Ne\(^{22}\), as a typical light nucleus, and Fe\(^{56}\) as the most abundant member of the iron group are indicated as “seed” nuclei at which synthesis begins. The capture path of the \( s \) process (slow compared with \( \beta^- \) decay) wanders along the stability line in the \( Z, A \) plane and finally terminates in alpha decay above bismuth. The \( r \) process (rapid compared to \( \beta^- \) decay) is caused by an intense neutron flux which drives the nuclear matter to the neutron-rich side of the stability line. For a temperature of \( 10^9 \) degrees the nuclei add neutrons until the neutron binding energy decreases to 2 Mev. At these “waiting points” \( \beta^- \) decay must occur before further neutrons can be added. Note the “staircase” effects at neutron magic numbers, \( N \), which is due to the difficulty, at this point, of adding the \( (N+1) \)th neutron. When the path thus moves closer to the stability line the \( \beta^- \) decay lifetime at the waiting points increases and the material builds to an excess abundance. Thus this effect at \( N = 50, 82, 126 \) leads to relatively sharp peaks in the abundance curve. These peaks occur with a smaller \( Z \) and \( A \) (proton-poor) than is associated with the stable nuclei with magic \( N \) and they are thus displaced 6 to 10 units in \( A \) below the magic peaks produced in the \( s \) process. The \( r \) process path is terminated by neutron-induced fission at \( A = 200 \) and the nuclear matter is fed back into the process at \( A \sim 108 \) and \( A \sim 146 \). Thus cycling and true steady flow occurs above \( A \sim 108 \).
given by their curves, because we are dealing with neutrons of higher energy than they were using (15 kev instead of \( \leq 1 \) kev).

(iii) For the few cross sections for which estimates are not available either from the thermal measurements or the reactivity measurements, we have interpolated from a smooth curve through the reactivity measurements for \( A > 40 \) and the thermal measurements for \( A < 20 \). In this interpolation we have maintained a ratio of \( \sim 3 \) for \( \sigma(\text{odd } A)/\sigma(\text{even } A) \) with a minimum at the isotopes with a magic number of 14 neutrons.

C. General Dynamics of the \( s \) and \( r \) Processes

In the buildup of nuclei by the \( s \) and the \( r \) processes the reactions which govern both the rate of flow and the track followed in the \( (A,Z) \) plane are the \( (n,\gamma) \) and \( (\gamma,n) \) reactions, beta decay, and, at the ends of the tracks, alpha decay in the case of the \( s \) process and neutron-induced fission in the case of the \( r \) process. We denote the rates of the \( (n,\gamma) \), \( (\gamma,n) \) and beta process as \( \lambda_n \), \( \lambda_\gamma \), \( \lambda_\beta \), where

\[
\lambda_n = 1/\tau_n = \sigma_n v_n n_n, \\
\lambda_\beta = 1/\tau_\beta = \text{const}/W_\beta^s, \\
\lambda_\gamma = 1/\tau_\gamma = \sigma_\gamma n_\gamma,
\]

and \( \sigma_n \) and \( \sigma_\gamma \) are the cross sections for the \( (n,\gamma) \) and \( (\gamma,n) \) reactions, respectively; \( v_n \) and \( n_n \) are the velocity and density of neutrons responsible for the \( (n,\gamma) \) reactions; \( n_\gamma \) is the density of \( \gamma \) radiation; and \( W_\beta \) is the beta-decay energy.

The general equation for the buildup of nuclei in the \( s \)-process is then

\[
dn(A,Z)/dt = \lambda_n (A-1, Z)n(A-1, Z) - \lambda_n (A,Z)n(A,Z) + \lambda_\beta (A, Z-1)n(A, Z-1) - \lambda_\beta (A,Z)n(A,Z) + \text{termination terms due to alpha decay at } A > 209. \tag{9}
\]

The general equation for the \( r \) process is

\[
dn(A,Z)/dt = \lambda_n (A-1, Z)n(A-1, Z) - \lambda_n (A,Z)n(A,Z) + \lambda_\beta (A, Z-1)n(A, Z-1) - \lambda_\beta (A,Z)n(A,Z) + \lambda_\gamma (A+1, Z)n(A+1, Z) - \lambda_\gamma (A,Z)n(A,Z) + \text{termination terms due to fission for } A \geq 260. \tag{10}
\]

For the \( s \) process, we have, in general,

\[\lambda_n < \lambda_\beta (\tau_n > \tau_\beta).\]

For the \( r \) process, we have

\[\lambda_n > \lambda_\beta (\tau_n < \tau_\beta).\]

As long as \( \lambda_n > \lambda_\beta \) buildup continues with \( Z \) constant and \( \lambda_n \) continuously decreases until \( \lambda_n (A,Z) \approx \lambda_\gamma (A+1, Z). \) At this point no further buildup can take place until beta decay occurs, thereby increasing \( Z \). The effective rate of neutron addition at this point is such that

\[\lambda_n (A,Z) - \lambda_\gamma (A+1, Z) < \lambda_\beta (A,Z).\]

The tracks of both the \( s \) and the \( r \) processes in the \( (A,Z) \) plane are shown in Figs. V.2 and V.3, and were obtained by methods described in the following sections.

VI. DETAILS OF THE \( s \) PROCESS

Excluding the alpha-decay terms, and remembering that \( \lambda_n < \lambda_\beta \), so that the beta decay takes place in a time short compared with neutron capture, we have from (9)

\[
dn(A)/dt = \lambda_n (A-1)n(A-1) - \lambda_\beta (A)n(A). \tag{11}
\]

The large \( \lambda_\beta \) terms in (11) serve only to produce the most stable \( Z \) at each \( A \). Equations (11) determine for the synthesis of the \( s \)-process nuclei as a function of time. In order to use this type of equation for calculation it is convenient to use the capture cross sections \( \sigma(A,Z) \) and to use, instead of \( t \), the number of neutrons injected per standard nucleus, since this is a number which can be calculated from the abundances of the nuclei in the neutron-producing reactions. Thus Eqs.
(11) can be rewritten (Fo55),
\[ \frac{dn(A)}{dn_o} = \frac{\sigma(A-1)n(A-1) - \sigma(A)n(A)}{\sum A \sigma(A)n(A)} \]
(12)

where \( n_o \) is the number of neutrons captured.

The properties of these equations can be thought of in terms of a hydrodynamical analogy. Consider a river bed with a number of depressions in it, each depression corresponding to a value of \( A \), and proportional in its volume to \( \sigma \). Suppose that water starts at \( A_i \) and flows in the direction of increasing \( A \). Water does not reach \( (A_i+2) \) until the depression at \( (A_i+1) \) has been filled. Similarly, water does not reach \( (A_i+3) \) until the depression at \( (A_i+2) \) is filled. Moreover, if there is a final reservoir at \( A_f \), water only reaches \( A_f \) when all of the depressions are filled. Thereafter steady flow, water going from \( A_i \) to \( A_f \) without change at intermediate points. This analogy is not complete since in the hydrodynamic case there is no flow at all beyond a particular depression until this depression has been filled, whereas in the nuclear case there is always a finite probability that some flow occurs.

The saturation or steady-flow abundance of a particular isotope is reached when \( \frac{dn(A)}{dn_o} = 0 \), i.e., when

\[ \frac{n(A)}{n(A-1)} = \frac{\sigma(A-1)}{\sigma(A)} \]

If a sufficient number of neutrons is available, the last of the isotopes in the \( s \) chain will achieve the full value of \( n(A) \) given by this equation and we shall have

\[ n(A) \sigma(A) = \text{constant} \]

over the range of isotopes built by the \( s \) process, apart from those immediately at the beginning of the chain in which the \( \sigma N \) product will decrease unless these nuclei are continuously augmented during the \( s \) process. However, if an insufficient number of neutrons is made available, we find that a plot of the \( n(A) \sigma(A) \) product is a monotonic function of \( A \), smoothly decreasing as \( A \) increases.

There are two separate regions in the abundance curve where a considerable proportion of the isotopes are made by the \( s \) process. The region \( 22 < A < 50 \) has been discussed previously (Fo55) and it was found that in this region steady flow was not attained, but that with a supply of neutrons such that \( n_o/n(\text{Ne}^{28}) = 2.8 \), the abundances of the isotopes built in the \( s \) chain up to \( \text{Tl}^{186} \) could be reasonably well reproduced. However, some modification of these results can now be made with the cross sections which are given in the appendix. A new plot of \( \sigma N \) against \( A \) is shown in Fig. VI.1. The effect of the improved cross sections is to show very clearly that \( \sigma N \) is truly a very smooth function of \( A \) for all of the isotopes built predominantly by the \( s \) process in the range of atomic weight \( 23 < A < 46 \), thus bearing out the correctness of our assignment of these nuclei.

One point of special interest is the case of scandium. This has only a single isotope, \( \text{Sc}^{48} \), and is made according to the appendix by the \( s \) process. However, when the \( \sigma N \) product is plotted by using the abundance given by Suess and Urey (Su56) the point falls below the curve delineated by the surrounding points. On the other hand, the solar abundance determined by Goldberg et al. (Go57) is larger than that given by Suess and Urey by a factor \( \sim 20 \), and this \( \sigma N \) product (which is the one plotted) lies above the curve. This strongly suggests to us that in the case of scandium the true abundance lies between the solar abundance and the estimate from terrestrial and meteoritic data. The points which lie above the curve, designated by crosses instead of filled circles, are due to the nuclei which have \( A = 24, 28, 32, 36, 40, 44, \) and \( 48 \). Their values of \( \sigma N \) are higher than those of their neighbors because their abundances are far larger. These are the nuclei which are built predominantly by the \( a \) process. The neutron-
capture cross sections, $\sigma$, clearly have no significance in their case.

Calculations similar to those carried out previously (Fo55) using the new cross section estimates have been carried out by Tuttle (private communication) and curves showing these results are also shown in Fig. VI.1. The best fit is obtained for $n_p/Ne^{22} = 2.1$, as compared with the value of $n_p/Ne^{22} = 2.8$ obtained in the original calculations. This new calculated curve fits the observed $\sigma N$ values only out to $A = 43$, whereas in the earlier calculations it appeared that a reasonable fit out to $A = 50$ was obtained. Thus, unless the cross section estimates in the region $45 < A < 50$ are considerably in error, we conclude from these new results that Ti$^{47}$, Ti$^{48}$, Ti$^{49}$, and Ti$^{50}$ are not built predominantly by the $s$ process, because their abundances are larger than we should expect on the basis of this theory. As described in Sec. III D, Ti$^{48}$ is built by the $\alpha$ process, and Ti$^{47}$, Ti$^{49}$, and Ti$^{50}$ have been tentatively ascribed to the $r$ process. This region of $A$ is the region where the $\alpha$, $s$, $\epsilon$, and $r$ processes all intersect, a point which was emphasized at the end of Sec. IV. Ti$^{48}$ is assigned partly to the $s$ process and partly to the $\epsilon$ process but its atomic abundance is higher than expected on this basis.

The second region in the abundance curve where many isotopes are made by the $s$ process is the region $63 \leq A \leq 209$. In this region, as described in Sec. II, assignments are more complicated because in many cases both the $s$ and the $r$ process contribute to the synthesis of an isotope. The correctness of our assignments can be tested by plotting both the $\sigma N$ products and also by plotting $N$ and $1/\sigma$ as a function of $A$. These plots are shown in Figs. VI.2 and VI.3. In Fig. VI.2, which shows the plots of $N$ and $1/\sigma$ for isotopes of odd $A$ only, the major peaks in both $N$ and $1/\sigma$ at magic neutron numbers $N = 50$ and $82$, the approach towards the peak at the double magic numbers $Z = 82$ and $N = 126$, and the minor peaks at $Z = 50$ and $A = 180$ are well defined. The run of the points throughout these curves shows that there are no major inconsistencies in our assignments. For a more detailed comparison we use the plot of $\sigma N$ against $A$ (Fig. VI.3). In this plot isotopes made solely by the $s$ process are distinguished from those in which the $s$ process and the $r$ process both contribute roughly equal amounts or in which the $s$ process dominates. In the latter two cases the $\sigma N$ product has been corrected by assuming that either $\frac{1}{2}$ or $\frac{3}{4}$ of the abundance is due to the $s$ process. In cases

![Fig. VI.2. The filled circles and diagonal crosses represent the abundances of even-$A$ and odd-$A$ isotopes, respectively, in the range of atomic weight $63 \leq A \leq 209$ which are made only in the $s$ process; open circles and vertical crosses represent abundances of even- and odd-$A$ isotopes, respectively, which are probably made in the $s$ process. The abundance scale is given on the left-hand ordinate. Two curves are drawn schematically through the even- and odd-$A$ points separately to show the trends of these abundances. The continuous curve which is also given is a plot of $\sigma N$, the reciprocal of the neutron capture cross section for isotopes of odd $A$, this scale being given by the right-hand ordinate. It will be seen that the peaks at the neutron magic numbers $N = 50$ and $82$, the proton magic number at $Z = 50$, and the rise to the doubly-magic peak at $N = 126$, $Z = 82$, together with the broader rise due to the spheroidal deformation effect near $A = 180$, are all shown by these curves, reflecting the effect that in local regions the $\sigma N$ product is constant, though over the whole range of $A$ which is displayed, the $\sigma N$ product shows considerable variation (cf. Fig. VI.3).]
where a smaller proportion than \( \frac{1}{3} \) of the abundance has been estimated to be made by the \( r \) process, no correction has been applied and the total \( \sigma N \) product has been plotted. The curve shows a steep decline from \( A \sim 63 \) to \( A \sim 100 \). Beyond this point, the slope appears to decrease more gradually, and it may tend to level off completely; however, the scatter of the points in this region, particularly beyond \( A \sim 170 \), makes the exact trend hard to determine. A large part of the scatter in this plot is most likely due to uncertainties in the estimated values of \( \sigma \) and also, to a lesser extent, uncertainties in the relative abundances of some elements in the rare earth region. Thus, in particular, the very low points at \( Ba^{134} \) and \( Pt^{195} \) are probably due in part to an underestimate of \( \sigma \), since as has previously been mentioned, no account has been taken of the fact that the lightest even isotope of an element will have a larger \( \sigma \) than heavier even isotopes. Again, the high values of \( \sigma N \) obtained for \( Sm^{148} \) and \( Sm^{150} \) may indicate that our estimates of all of the \( \sigma 's \) for the samarium isotopes have been consistently too high.

The relative abundances of the isotopes of barium show in a very striking way how the \( s \)-process operates. The light isotopes \( Ba^{130} \) and \( Ba^{132} \) cannot be made on the \( s \) or \( r \) processes and are indeed very rare with atomic abundances 0.0037 and 0.00356, respectively. The fact that they are almost equal can be well understood on the basis of their production in the \( p \) process (cf. Sec. IX). They result from the decay of positron-emitting progenitors at 130 and 132, made in equal abundance in the \( p \) process. On the other hand, the isotopes from \( Ba^{134} \) to \( Ba^{138} \) show much greater abundances, with a marked increase with increasing number of neutrons. \( Ba^{136} \) and \( Ba^{138} \) can only be made in the \( s \) process, \( Ba^{136} \) is magic in this process, and \( Ba^{135} \) and \( Ba^{137} \) are predominantly made in the \( s \) process. The marked rise in abundances from \( Ba^{134} \) to \( Ba^{138} \), i.e., 0.0886, 0.241, 0.286, 0.414, 2.622 can be understood on the basis of the decreasing neutron capture cross section. This is to be expected as more neutrons are added, and culminates in the very low cross section and large abundance expected for \( Ba^{136} \) with magic \( N = 82 \). Our estimates of cross sections in the appendix do not take the gradual rise with \( N \) into account, but it is strongly indicated by the Oak Ridge results. It will be interesting to see if neutron activation measurements on \( Ba^{134} \) (producing 29 hr \( Ba^{134m} \)), and on \( Ba^{136} \) (producing 2.6 min \( Ba^{137m} \)), bear out the ratios expected relative to \( Ba^{138} \). The expected cross sections are \( 11.4 \times 2.622/0.0886 = 336 \) mb for \( Ba^{134} \) and \( 11.4 \times 2.622/0.286 = 109 \) mb for \( Ba^{136} \). Unfortunately, the isomeric states may be produced with only a small fraction of these cross sections and this will lead to some ambiguity in interpreting the experimental result. Unambiguous results would be obtained by measuring the total absorption cross sections, but this is very difficult experimentally. It is our view, however, that such measurements would serve as a crucial test of the validity of the \( s \) process.

The steep part of the curve shown in Fig. VI.3, followed by part which tends to level off, suggests that the curve has a composite character and that the abundances in the solar system of \( s \)-process isotopes
have been produced in more than one stellar synthesis process. Thus, the simplest possibility is that one process is responsible for the part of the curve in the range 63 < A ≤ 100 and that this part is similar in shape to the \( r \nu \) plot for the isotopes in the range 33 < A < 46, shown in Fig. VI.1, falling to zero very rapidly beyond \( A \sim 100 \). This part of the curve shows that there have not been enough neutrons available per Fe\(^{56}\) nucleus to build the nuclei to their saturation abundances. Thus, it may be that in this region the \( C^4(\alpha,\gamma)O^{16} \) reaction has been the neutron source. A second process may be responsible for that portion of the curve beyond \( A \sim 100 \). In this case, since this latter slope is small or zero, this is strongly suggestive of steady flow being achieved and of all of these nuclei reaching their saturation abundances. The rates at which the neutrons are released by \( C^4(\alpha,\gamma)O^{16} \) and \( Ne^{23}(\alpha,\gamma)Mg^{24} \) are discussed in Sec. III. The time-scales obtained from arguments based on particular isotopes in the s-process chain are given in Sec. II, where it was shown that these time-scales were compatible with the result that the \( \sigma N \) product falls steeply to \( A \sim 100 \), implying a paucity of neutrons, and then levels off, the abundances reaching their saturation values. Thus at the end of the s-process chain we expect that cycling among the lead isotopes, following the alpha decays at Pb\(^{208}\), Bi\(^{209}\), and Bi\(^{210}\) which are shown in the appendix and illustrated in Figs. V.2 and V.3 has taken place, so that the lead isotopes which are included in the cycle, Pb\(^{206}\), Pb\(^{207}\), and Pb\(^{208}\), have been built up to a far greater extent than Pb\(^{204}\) which is not in the cycle. Some lead is also produced by the \( r \) process, and if this contribution is taken into account we can calculate the amount of cycling which has taken place, and also predict a total abundance of lead. This question is discussed in Sec. VIII, following detailed explanation of \( r \)-process dynamics.

If this curve in Fig. VI.3 is a composite of two curves, it is also clear that the degrees of dilution of the abundances in the two parts are different, since larger over-abundance ratios are demanded to obtain complete saturation than if saturation is not achieved. Such a situation is entirely possible, since the two different processes might have occurred in two different red-giant stars, in which the dilution was a function both of the original composition, the mixing to the surface, and the ejection into the interstellar medium. Alternatively, they might have occurred in a single star at different evolutionary stages in which the structures and hence these same parameters would be changed. To study this question further it is important to investigate the composition of a star in which it is believed that the \( r \) process has recently been occurring. The S-type stars and the so-called Ba II stars fall into this category, and the results of such an investigation are described in Sec. XI D.

Finally, in Table VI.1 we show the \( \sigma N \) products for just those isotopes for which experimental measurements of cross sections at "25-kev" neutron energies have been made at Oak Ridge (La57b) and at Livermore (Bo57), and which have been assigned to the \( r \) process or in which the contributions of the \( \nu \) process have been estimated. Five of the isotopes lie in the range 23 < A ≤ 46, while the remainder have A ≥ 63. This table shows the same effects as are shown in Figs. VI.1 and VI.3.

<table>
<thead>
<tr>
<th>( \sigma ) (mb)</th>
<th>N(Si = 10)</th>
<th>( \sigma N )</th>
<th>Assignment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Na(^{23})</td>
<td>1</td>
<td>4.38 \times 10^{10}</td>
<td>4.38 \times 10^{10}</td>
</tr>
<tr>
<td>Mg(^{24})</td>
<td>&lt;14</td>
<td>1.00 \times 10^{10}</td>
<td>&lt;1.4 \times 10^{10}</td>
</tr>
<tr>
<td>Al(^{27})</td>
<td>1.4</td>
<td>9.48 \times 10^{10}</td>
<td>1.3 \times 10^{10}</td>
</tr>
<tr>
<td>Cl(^{37})</td>
<td>1.1</td>
<td>2180</td>
<td>2.4 \times 10^{10}</td>
</tr>
<tr>
<td>K(^{40})</td>
<td>19</td>
<td>219</td>
<td>4.2 \times 10^{10}</td>
</tr>
</tbody>
</table>

\( \nu \) process.

VII. DETAILS OF THE \( \nu \) PROCESS

The essential feature of the \( \nu \) process is that a large flux of neutrons becomes available in a short time interval for addition to elements of the iron group, or perhaps, in cases where the abundances in the iron group are abnormally small, for addition to light nuclei such as Ne\(^{23}\).

From the point of view of the present section, the precise source of the neutrons is not important; any source capable of supplying a large neutron flux on a short time-scale of order 10–100 sec, would meet the requirements. In Sec. III F we discussed the possibility that a neutron density of 10^{23}/cm^{3} and a flux of 4X10^{21}/cm^{3} sec might be produced in supernova envelopes at temperatures \( \sim 10^{9} \) degrees. We assume these conditions in the following discussion of the path of the \( \nu \) process and of the abundances produced by it.

Fong (Fo57), using a non-dynamical analogy, has previously calculated abundances produced by an \( \nu \) process. Our method is somewhat different in that we include the (\( \gamma,\nu \)) as well as (\( n,\gamma \)) and \( \beta^- \) decay in our calculations.